MATHEMATICS

First Preparatory - Second Term   2024 - 2025

Student's Book

Prepared By

Mr.
Osama Zaid Sharif

Prof. Dr.
Samar Abdel Fattah Lasheen

Ass. Prof. Dr.
Amal El-Shahat Hafez

Mr.
Magdy Abdel Fattah El-Safty

Mr.
Ibrahim Abdel Latif El-Saghir

Mr.
Bassem Magdy Roushdy

Mr.
Islam Mohamed Ibrahim

Mr.
Sameh Labib Attia

Name : ....

Class : ....

School : ....

منهاجي
متعة التعليم الهادف

Introduction

In the framework of the continuous development of the education system, and as part of the development plan that began with the kindergarten stage and continues until the end of the secondary stage, according to Egypt's Vision 2030 and the Ministry of Education's development plan, this book has been prepared to complete and implement the development processes of educational curriculums for the preparatory stage to create a qualitative leap in the way Egypt's students are prepared to deal with future challenges.

We are pleased to present this mathematics textbook for the first preparatory grade for the second term, which includes a set of tasks and performances that help students solve mathematical problems. This aims to achieve learning outcomes that keep pace with the challenges of the 21st century, whose structure is reinforced by the rapid revolution in information and communication technology.

The current curriculum aims to bring about a qualitative leap in the teaching and learning of mathematics. It also aims to provide an appropriate means to achieve the general educational goals in an integrated manner that aligns with the following:

• Emphasizing the positive role of the student in the learning process.

• Included knowledge curriculum, skills, values, positive trends needed for education, good citizenship, productive work and active participation in sustainable development programs.

• Including of modern positive trends in curriculum building, such as critical thinking skills, problem-solving skills, self-learning skills, collaborative learning, and effective communication with knowledge sources.

• Developing performance skills by focusing on self-learning and collaborative work.

• Achieving integration between mathematics and other subjects across different educational stages.

• Providing students with the opportunity to choose activities that suit their abilities, tendencies, and needs.

Asking Almighty God to make this book beneficial and hope that it will be part of a great effort to elevate Egypt to the ranks of advanced countries, ensuring a great future for all students. We wish you a very successful academic year full of achievements and success.

May God grant success,
The Authors

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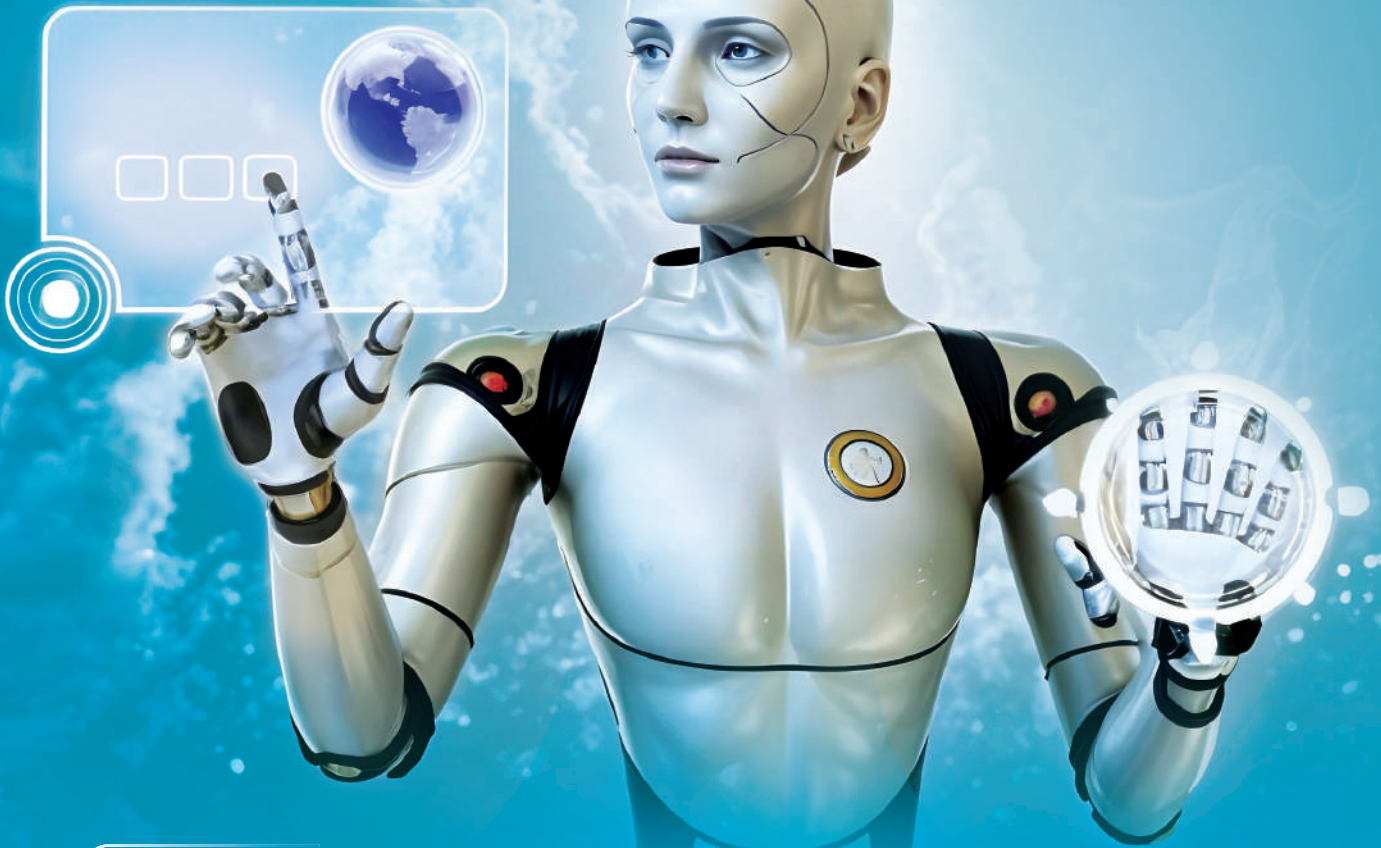## First Preparatory - Second Term

UNIT 1

Powers, Exponents, and Roots

Unit Lessons

1-1 Powers and Exponents

1-2 Scientific Notation

1-3 Square Roots and Cube Roots

Artificial Intelligence (AI) is used in the development of interactive applications to enhance educational processes.

▶ **Can Artificial Intelligence (AI) mechanisms be developed to perform complex calculations involving exponents and roots?**

Issues and Life Skills

- Mathematical Communication
- Information Technology
- Critical Thinking
- Mathematical Understanding
- Creative Thinking

Values

- Respect
- Perseverance
- Justice
- Responsibility
- Belonging

■ Learning Outcomes

- Learn the concept of repeated multiplication and exponential form
- Distinguish between the concepts of power and exponent
- Apply the laws of exponents in solving exercises
- Use the laws of exponents to simplify mathematical expressions
- Use the positive, negative, and zero exponents in solving exercises

■ Vocabulary

- Repeated Multiplication
- Exponent
- Power
- Base

■ Think with your classmates

- Is $2^5 = 5^2$?
- Is $2^5 = 2 \times 5$?

■ Note that

$2^5 = 32$

2^5 is the exponential form of the number 32

Get Ready!

Euglena is a unicellular organism that reproduces by binary fission, the first division results in two cells which in turn produce four cells after the second division, and so on.

What is the number of cells produced after 8 divisions?

In this lesson, you will learn the concept of exponents and how to calculate the powers of a number, as well as how to use the laws of exponents to solve exercises, enabling you to solve such problems.
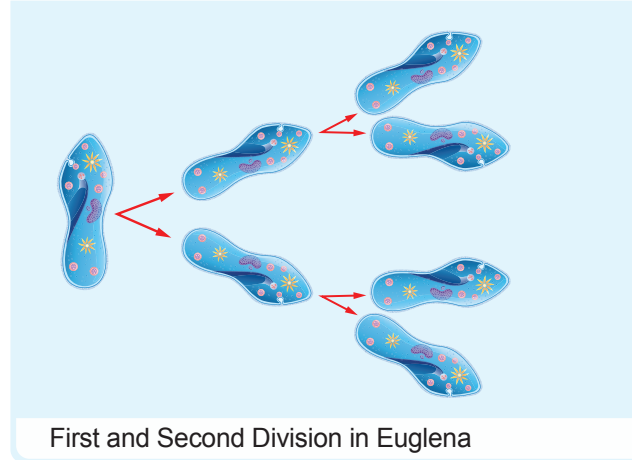
First and Second Division in Euglena

Think & Discuss!

What is the value of $2^7 \times 2^5$?

Marwan, Mariam, and Mohammed all answered the previous question, and their answers were as follows:

Marwan	**Mariam**	**Mohammed**
$2^7 \times 2^5 = 4^{35}$	$2^7 \times 2^5 = 2^{35}$	$2^7 \times 2^5 = 2^{12}$

Which of them is correct? Discuss.

Learn!

Repeated Multiplication and Exponential form

The result of multiplying repeated factors can be expressed using the powers or exponential form, that has an exponent and a base.

Example:

The exponent indicates the number of times the base is used as a factor

A factor repeated 5 times
$$2 \times 2 \times 2 \times 2 \times 2 = 2^5$$

The base is the repeated factor

2^5 is read as "2 raised to the power of 5" or "2 to the power of 5", meaning "2 multiplied by itself 5 times".

In general

Factor repeated n times

If n is a positive integer, then for any number a: $a \times a \times a \times ... \times a = a^n$

Example 1

Write each of the following using exponents:

❶ $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$ 　　　　❷ $(-x) \times (-x) \times (-x) \times (-x)$

❸ $\frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3}$ 　　　　❹ $3 \times 3 \times a \times a \times a \times 3 \times 3$

✔️

① $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^7$

② $(-x) \times (-x) \times (-x) \times (-x) = (-x)^4$

③ $\frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} = \left(\frac{2}{3}\right)^5$

④ $3 \times 3 \times a \times a \times a \times 3 \times 3 = 3 \times 3 \times 3 \times 3 \times a \times a \times a = 3^4 \times a^3$
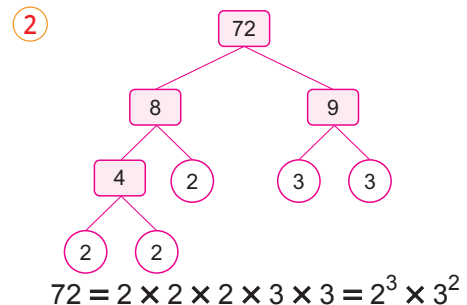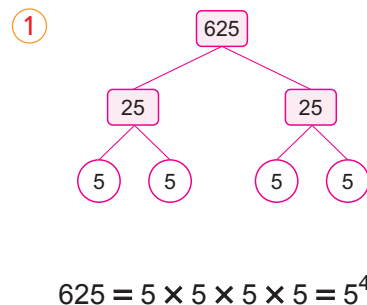
Example 2

Write each of the following in exponential form such that the base is a prime number:

❶ 625 　　　　　　　　　　　　❷ 72

✔️

Factorize each number into its prime factors as follows:

① 625

❷ 72

$$625 = 5 \times 5 \times 5 \times 5 = 5^4$$ 　　　$$72 = 2 \times 2 \times 2 \times 3 \times 3 = 2^3 \times 3^2$$

Example 3

If $a = 2$, $b = -5$, find the numerical value of each of the following:

❶ $3b^2$ 　　❷ $(3b)^2$ 　　❸ $a^3 + b^3$ 　　❹ $(a+b)^3$

✔️

① $3b^2 = 3 \times (-5)^2 = 3 \times 25 = 75$

② $(3b)^2 = (3 \times (-5))^2 = (-15)^2 = 225$

③ $a^3 + b^3 = 2^3 + (-5)^3 = 8 + (-125) = -117$

④ $(a+b)^3 = (2 + (-5))^3 = (-3)^3 = -27$

■ **Technology**

You may use a calculator to perform calculations to verify your answers

Even and Odd Exponents of a Negative Base

Note that : • $(-5)^2 = (-5) \times (-5) = 25 > 0$

• $(-5)^3 = (-5) \times (-5) \times (-5) = -125 < 0$

In general

• When the base is a negative number and the exponent is an even number, the result is a positive number.

• When the base is a negative number and the exponent is an odd number, the result is a negative number.

① Isslam wrote that
$5^2 \times 5^3 = 25^5$
Is it true what Isslam wrote? Discuss.

② Ahmed wrote that
$5^2 \times 5^3 = 5^5$
Is it true what Ahmed wrote? Discuss.

③ Samar wrote that
$5^2 \times 5^3 = 5^6$
Is it true what Samar wrote? Discuss.

④ Amal wrote that
$5^2 \times 5^3 = 25^6$
Is it true what Amal wrote? Discuss.

■ **Think & Discuss**

① Magdy wrote that
$\dfrac{3^8}{3^2} = 1^6$
Is it true what Magdy wrote? Discuss.

② Ibrahim wrote that
$\dfrac{3^8}{3^2} = 3^4$
Is it true what Ibrahim wrote? Discuss.

③ Bassem wrote that
$\dfrac{3^8}{3^2} = 3^6$
Is it true what Bassem wrote? Discuss.

Multiplication and Division of Powers with the Same Base

First Multiplication Law

$$2^3 \times 2^4 = \underbrace{(2 \times 2 \times 2)}_{\text{3 Factors}} \times \underbrace{(2 \times 2 \times 2 \times 2)}_{\text{4 Factors}} = 2^7$$
$$\underbrace{}_{\text{7 Factors}}$$

To multiply powers with the same base, keep the base and add the exponents.

In general

For any rational number a, and two integers m, n:

$$a^m \times a^n = a^{m+n}$$

For example:

• $3^3 \times 3^2 = 3^{3+2} = 3^5$ • $(-2)^7 \times (-2) = (-2)^{7+1} = (-2)^8$

Generalization :

The multiplication law can be generalized to more than two powers as long as they have the same base.

For example: $10^2 \times 10^3 \times 10 = 10^{2+3+1} = 10^6$

Second Division Law

$$\frac{2^7}{2^4} = \frac{\overbrace{\cancel{2} \times \cancel{2} \times \cancel{2} \times \cancel{2} \times 2 \times 2 \times 2}^{\text{7 Factors}}}{\underbrace{\cancel{2} \times \cancel{2} \times \cancel{2} \times \cancel{2}}_{\text{4 Factors}}} = \overbrace{2 \times 2 \times 2}^{\text{3 Factors}} = 2^3$$

To divide powers with the same base, keep the base and subtract the exponents.

In general

For any rational number a, not equal to zero, and two integers m, n:

$$\frac{a^m}{a^n} = a^{m-n}$$

For example: • $\dfrac{3^5}{3^3} = 3^{5-3} = 3^2$ • $\left(\dfrac{1}{2}\right)^8 \div \left(\dfrac{1}{2}\right)^6 = \left(\dfrac{1}{2}\right)^{8-6} = \left(\dfrac{1}{2}\right)^2$

Example 4

Find in simplest form the result of each of the following:

① $\dfrac{6^7 \times 6^2}{6^3 \times 6^5}$ ② $\dfrac{(-2)^7 \times 3^6}{(-2)^5 \times 3^4}$ ③ $\dfrac{(-a)^4 \times a^6}{(-a)^5 \times (-a)^3}$

✓

■ **Another Solution**

① $\dfrac{6^7 \times 6^2}{6^3 \times 6^5} = \dfrac{6^{7+2}}{6^{3+5}} = \dfrac{6^9}{6^8} = 6^{9-8} = 6^1 = 6$

$\dfrac{6^7 \times 6^2}{6^3 \times 6^5} = 6^{7+2-3-5}$
$= 6^1 = 6$

✅

$$② \quad \frac{(-2)^7 \times 3^6}{(-2)^5 \times 3^4} = (-2)^{7-5} \times 3^{6-4} = (-2)^2 \times 3^2$$
$$= 4 \times 9 = 36$$

$$③ \quad \frac{(-a)^4 \times a^6}{(-a)^5 \times (-a)^3} = \frac{a^4 \times a^6}{-a^5 \times -a^3} = \frac{a^4 \times a^6}{a^5 \times a^3} = \frac{a^{4+6}}{a^{5+3}}$$
$$= \frac{a^{10}}{a^8} = a^{10-8} = a^2$$

Example 5

Technology: In the field of computer technology, the byte is one of the units used to measure file size. If you know that one kilobyte equals 2^{10} bytes, and one gigabyte equals 2^{30} bytes, **how many kilobytes does one gigabyte contain?**

✅

$$\text{Gigabyte} = \frac{2^{30}}{2^{10}} \text{ kilobytes}$$
$$= 2^{30-10} \text{ kilobytes}$$
$$= 2^{20} \text{ kilobytes}$$

■ Diversified Strategies

If $a \neq 0$
From the multiplication law of powers, you can find that:

❶ $a^n \times a^0 = a^{n+0} = a^n$
It can be deduced that a^0 is the multiplicative identity i.e. $\boxed{a^0 = 1}$

❷ $a^n \times a^{-n} = a^{n-n}$
$= a^0 = 1$
i.e. a^n and a^{-n} are each the multiplicative inverse of the other.
Therefore: $\boxed{a^{-n} = \frac{1}{a^n}}$

■ Critical Thinking

❶ Is $5^2 \times 5^{-3}$ equivalent to $5^3 \times 5^{-2}$?

❷ State the difference between $(-5)^2$ and 5^{-2}

Zero Exponent and Negative Integer Exponents

2^3	2^2	2^1	2^0	2^{-1}	2^{-2}	2^{-3}
8	4	2	1	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$

$\div 2 \quad \div 2 \quad \div 2 \quad \div 2 \quad \div 2 \quad \div 2$

By observing the pattern in the previous table, you can find that:

$\boxed{2^0 = 1}$ $\boxed{2^{-1} = \frac{1}{2}}$ $\boxed{2^{-2} = \frac{1}{4} = \frac{1}{2^2}}$ $\boxed{2^{-3} = \frac{1}{8} = \frac{1}{2^3}}$

In general

① Any number, not equal to zero, raised to the power of zero is equal to 1
i.e. For any number $a \neq 0$, $\boxed{a^0 = 1}$

For example: $3^0 = 1$, $\left(\frac{2}{3}\right)^0 = 1$, $(-2)^0 = 1$, $\left(-\frac{1}{4}\right)^0 = 1$

② Any number, not equal to zero, raised to the power of $(-n)$ is equal to the multiplicative inverse of the same number raised to the power of n
i.e. For any number $a \neq 0$, $\boxed{a^{-n} = \frac{1}{a^n}}$

For example: $5^{-2} = \frac{1}{5^2}$, $\left(\frac{1}{2}\right)^{-3} = 2^3$, $\left(\frac{3}{7}\right)^{-1} = \frac{7}{3}$

Note that division by zero is undefined, and thus, when there are symbols in the denominator, these symbols must not equal to zero.

Example ⑥

Simplify each of the following to its simplest form:

① $\dfrac{2^5 \times 2^{-2}}{2^3 \times 2^{-4}}$ ② $\dfrac{x^{-6} \times x^{-2}}{x^{-3} \times x^{-4}}$

✓

① $\dfrac{2^5 \times 2^{-2}}{2^3 \times 2^{-4}} = \dfrac{2^{5-2}}{2^{3-4}} = \dfrac{2^3}{2^{-1}} = 2^{3-(-1)} = 2^4 = 16$

② $\dfrac{x^{-6} \times x^{-2}}{x^{-3} \times x^{-4}} = \dfrac{x^{-6+(-2)}}{x^{-3+(-4)}} = \dfrac{x^{-8}}{x^{-7}} = x^{-8-(-7)}$

$= x^{-8+7} = x^{-1} = \dfrac{1}{x}$

Lesson Assessment

😊 🙂 😐 ☹️

First **Measuring Conceptual Understanding**

▶ **Choose the correct answer from the given ones:**

① Which of the following equals
$3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$?
(a) 3×7 (b) 7^3 (c) 3^7 (d) $3 + 7$

② Which of the following equals $(-2)^3$?
(a) -6 (b) 6 (c) 8 (d) -8

③ Which of the following equals -3^4?
(a) -12 (b) -7 (c) 81 (d) -81

④ Which of the following equals 2^{-4}?
(a) -16 (b) 16 (c) $\dfrac{1}{8}$ (d) $\dfrac{1}{16}$

⑤ Which of the following equals $a^{-1} \times a^3$?
(a) a^2 (b) a^4 (c) $\dfrac{1}{a^2}$ (d) $\dfrac{1}{a^3}$

⑥ If $2^4 \times a = 2^{20}$, what is the value of a?
(a) 2^{24} (b) 2^{16} (c) 2^5 (d) 2

⑦ If $2^{-5} \times a = 1$, what is the value of a?
(a) 5^2 (b) 2^5 (c) 2^{-5} (d) 5^{-2}

⑧ Which of the following is the additive inverse of the number 4^{-3}?
(a) $(-4)^3$ (b) $(-4)^{-3}$ (c) 4^3 (d) 4^{-3}

⑨ Which of the following expresses $\dfrac{y^{-2}}{y^{-6}}$ in its simplest form?
(a) y^4 (b) $\dfrac{1}{y^4}$ (c) y^8 (d) $\dfrac{1}{y^8}$

⑩ $5\,a^0 - (5\,a)^0 = $ ············
(a) 0 (b) 4 (c) 5 (d) 10

Second **Applying Scientific Concepts**

▶ **Find the numerical value of each of the following expressions at the given values:**

⑪ $(-b)^a$ at $a = 3$, $b = -5$

⑫ $a^2 \times b^{-2}$ at $a = 5$, $b = -3$

⑬ $a\,(b - c^d)$ at $a = 2$, $b = 3$, $c = 6$, $d = 3$

▶ **Write each of the following using positive exponents:** ⑭ 8^{-4} ⑮ $\dfrac{1}{10^{-4}}$

▶ **Simplify each of the following to its simplest form:**

⑯ $\dfrac{x^7 \times x^{11}}{x^3 \times x^5}$

⑰ $\dfrac{(-4)^4 \times (-4)^3 \times 4^2}{(-4)^6 \times (-4)^5}$

⑱ $\dfrac{-3 \times 5^{-3} \times 2^5}{2^3 \times 3^{-1} \times 5^{-4}}$

⑲ $\dfrac{a^{-1} \times a^2 \times a^{-3}}{a^4 \times a^{-7}}$

▶ **By using prime factors and exponents, write each of the following numbers:**

⑳ 125   ㉑ 324

▶ **Find the missing exponent in each of the following:**

㉒ $a^{\square} \times a^7 = a^9$ ㉓ $\dfrac{b^{\square}}{b^4} = b^3$ ㉔ $q^{-3} \times q^{\square} = 1$ ㉕ $\dfrac{d^{-3}}{d^{\square}} = d$

▶ **Choose the correct answer from the given ones:**

㉖ Which of the following equals $2^a + 2^a$?

(a) 4^{2a} (b) 2^a (c) 2^{a+1} (d) 2^{2a}

㉗ Which of the following equals a third of the number 3^x?

(a) 1^x (b) $\left(\dfrac{1}{3}\right)^x$ (c) 3^{x+1} (d) 3^{x-1}

Third   **Analysis and Subjects Integration**

㉘ **Statistics:** Without evaluating each value, find the median for the numbers:

2^0 , 2^{-3} , 2^{-1} , 2^{-4} , 2^3

㉙ **Astronomy:** If the mass of the sun is approximately 10^{27} tons, what would be the mass of the sun in kilograms? (Write the result in exponential form with base 10)

The reliance on renewable and clean energy sources such as solar energy must be increased to combat the phenomenon of global warming.

Creative Thinking

㉚ Write the numbers 2, 0, 2, 3 in the following squares to get the largest possible value for the numerical expression.

$$\square^{\square} \times \square^{\square}$$

㉛ If a and b are two positive integers such that $a^b = 81$, find the smallest value for the expression $a + b$.

Evaluate *your understanding!*

How well do you understand powers and exponents? Tick the right box.

Scientific Notation

- Express numbers using scientific notation
- Convert numbers from standard form to scientific notation and vice versa
- Compare and order a set of numbers in scientific notation
- Perform arithmetic operations on numbers in scientific notation

.

Get Ready!

Both Osama and Mustafa wanted to know the distance between planet Earth and the Sun.

Osama found in one of the astronomy books that the average distance between the Earth and the Sun is 149,600,000 kilometers, while Mustafa, through one of the artificial intelligence applications, discovered that the average distance between them is 1.496×10^8 kilometers.

149,600,000 kilometers

Did both of them get the same result?

In this lesson, you will learn how to express very large or very small numbers in scientific notation and perform operations on them; this will enable you to solve such problems.

■ **Vocabulary**

- Scientific Notation
- Standard Form
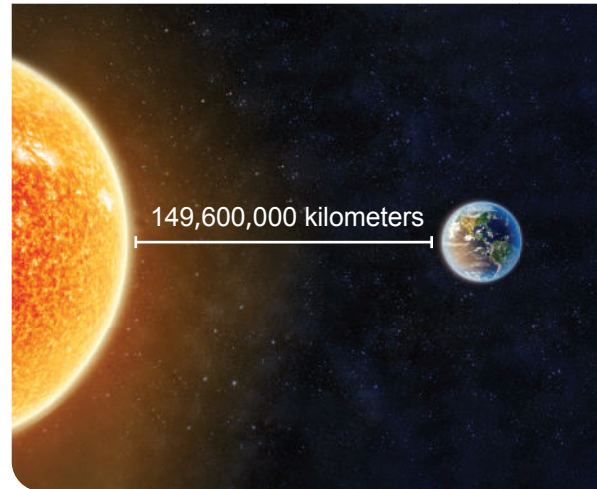
Think & Discuss!

When you type the number $\dfrac{3}{200,000,000}$

on the scientific calculator, you find that the number appears on the screen in the form 1.5×10^{-8} What is the benefit of writing numbers in this form that appeared on the screen?

1.5×10^{-8}

■ **Think**

Why each of the following numbers is not in scientific notation?

1. 64×10^5
2. $2.1 \times 10^{2.5}$
3. 0.82×10^{-4}

Learn!

Scientific Notation

Scientific notation is a method of writing very large or very small numbers, in which the number is expressed as the product of two factors, one of them has an absolute value greater than or equal to 1 and less than 10, and the other is 10 raised to an integer power.

i.e. The number is expressed in scientific notation as:

$$a \times 10^n \text{ where } 1 \leq |a| < 10, \, n \in Z$$

For example:

Each of the following numbers is written in scientific notation:

- 4.8×10^9
- -2.56×10^{-4}
- 1×10^{18}

■ **Self-Evaluation** ①

Write each of the following in scientific notation:

1 192,000,000
2 0.00000164
3 12 million
4 75×10^5

Writing numbers in scientific notation

Example 1

Write each of the following in scientific notation:

1 4,650,000   2 0.00079   3 571×10^9

✓

1 **4 6 5 0 0 0 0 . 0**

The decimal point moves 6 places to the left where $(1 \leq 4.65 < 10)$
Then multiply the result by 10^6

Thus, $4,650,000 = 4.65 \times 10^6$

2 **0 . 0 0 0 7 9**

The decimal point moves 4 places to the right where $(1 \leq 7.9 < 10)$
Then divide the result by 10^4 (i.e., multiply by 10^{-4})

Thus, $0.00079 = 7.9 \times 10^{-4}$

3 $5\ 7\ 1\ . \times 10^9$

The decimal point moves 2 places to the left, then multiply the result by 10^2

Thus, $571 \times 10^9 = 5.71 \times 10^2 \times 10^9 = 5.71 \times 10^{11}$

■ **Technology**

Use a scientific calculator to write each of the previous numbers in scientific notation to verify the results.

■ **Think with your classmates**

How old are you in minutes? Write the result in scientific notation.

Writing numbers in standard form

To convert the number $a \times 10^n$ from scientific notation to standard form:
- When n is positive, move the decimal point n places to the right.
- When n is negative, move the decimal point $|n|$ places to the left.

Example 2

Write each of the following in standard form:

1 2.4×10^4 2 8.95×10^{-2}

✓

1 Move the decimal point (4 places) to the right because the exponent is (4) after placing zeros to the right

$2.4 \times 10^4 = 24000$

2 Move the decimal point (2 places) to the left because the exponent is (-2) after placing zeros to the left

$8.95 \times 10^{-2} = 0.0895$

■ **Enriched Information**

• The solar system contains eight planets, the largest one is Jupiter and the smallest is Mercury.

• Use the internet to find the masses of the remaining planets in the solar system, write them in scientific notation, and then arrange all of them from the smallest to the largest.
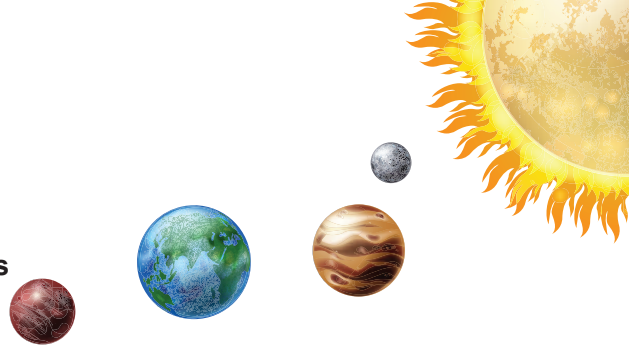
Example 3

Astronomy: Arrange the masses of the following planets in an ascending order.

Planet	Mars	Earth	Mercury	Venus
Mass in kilograms	6.4×10^{23}	5.97×10^{24}	3.3×10^{23}	4.87×10^{24}

Step One: If the masses are written in scientific notation, the smaller exponent indicates the smaller number, thus:

$$\begin{array}{ll} \text{Mercury} & 3.3 \times 10^{23} \\ \text{Mars} & 6.4 \times 10^{23} \end{array} \quad < \quad \begin{array}{ll} \text{Venus} & 4.87 \times 10^{24} \\ \text{Earth} & 5.97 \times 10^{24} \end{array}$$
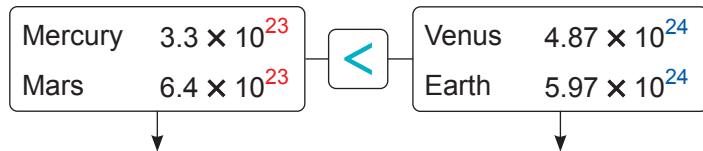
Step Two: **When the exponents are equal, $3.3 < 6.4$**

$3.3 \times 10^{23} < 6.4 \times 10^{23}$

When the exponents are equal, $4.87 < 5.97$

$4.87 \times 10^{24} < 5.97 \times 10^{24}$

Thus, $3.3 \times 10^{23} < 6.4 \times 10^{23} < 4.87 \times 10^{24} < 5.97 \times 10^{24}$

Therefore, Mass of Mercury < Mass of Mars < Mass of Venus < Mass of Earth

▸ **Attempt to solve the example in another way by unifying the exponents.**

Operations on numbers in scientific notation

Writing very large or very small numbers in scientific notation makes the calculations on them easier.

Example 4

Find the result of each of the following in scientific notation:

❶ $(2.5 \times 10^6) \times (6 \times 10^{12})$

❷ $(1.5 \times 10^{-7}) \div (0.5 \times 10^{-5})$

❸ $(2.1 \times 10^4) + (4.1 \times 10^5)$

❹ $(3.21 \times 10^{13}) - (8.1 \times 10^{12})$

① $(2.5 \times 10^6) \times (6 \times 10^{12}) = (2.5 \times 6) \times (10^6 \times 10^{12})$

$$= 15 \times 10^{18} = 1.5 \times 10^{19}$$

② $\dfrac{1.5 \times 10^{-7}}{0.5 \times 10^{-5}} = \left(\dfrac{1.5}{0.5}\right) \times \left(\dfrac{10^{-7}}{10^{-5}}\right) = 3 \times 10^{-2}$

▪ Note that

To make the process of addition or subtraction easier, unify the exponents of number 10.

③ $(2.1 \times 10^4) + (4.1 \times 10^5) = (2.1 \times 10^4) + (41 \times 10^4)$
$$= (2.1 + 41) \times 10^4 \quad \text{"Distributive property"}$$
$$= 43.1 \times 10^4 = 4.31 \times 10^5$$

④ $(3.21 \times 10^{13}) - (8.1 \times 10^{12}) = (32.1 \times 10^{12}) - (8.1 \times 10^{12})$
$$= (32.1 - 8.1) \times 10^{12} \quad \text{"Distributive property"}$$
$$= 24 \times 10^{12} = 2.4 \times 10^{13}$$

Collaborative Activity

Collaborate with one of your friends and use the internet to find the areas of the continents of the world in square kilometers.

Write these areas in scientific notation, and ask your friend to arrange them from the smallest area to the largest area.

Lesson Assessment

First — Measuring Conceptual Understanding

▶ **Choose the correct answer from the given ones:**

① Which of the following numbers is written in scientific notation?
- (a) $1.5 \times 10^{4.5}$
- (b) 31.5×10^5
- (c) 15×10^5
- (d) 3.15×10^5

② Which of the following numbers is not in scientific notation?
- (a) 2.35×10^7
- (b) 23.5×10^6
- (c) 2.35×10^{-7}
- (d) 3.5×10^{-6}

③ Which of the following expresses the number 8 million in scientific notation?
- (a) 8×10^7
- (b) 8×10^6
- (c) 8×10^{-6}
- (d) 8×10^8

④ Which of the following equals 0.000073?
- (a) 7.3×10^6
- (b) 7.3×10^5
- (c) 7.3×10^{-5}
- (d) 7.3×10^{-6}

⑤ If $6.3 \times 10^n = 0.00063$, what is the value of n?
- (a) -4
- (b) -3
- (c) 3
- (d) 4

⑥ If the number $y \times 10^{-9}$ is written in scientific notation, which of the following could be the value of y?
- (a) 0.6
- (b) 6
- (c) 60
- (d) 600

Second — Applying Scientific Concepts

▶ **Choose the correct answer from the given ones:**

⑦ If $39 \times 10^{-8} = k \times 10^{-7}$, what is the value of k?
- (a) 39
- (b) 3.9
- (c) 0.39
- (d) 0.039

⑧ If the speed of light is equal to 300,000 km/s, what is the speed of light in m/s?
- (a) 3×10^5
- (b) 3×10^7
- (c) 3×10^8
- (d) 3×10^{10}

▶ **Write each of the following in scientific notation:**

⑨ 24,500,000 ⑩ 0.000049 ⑪ 9 million ⑫ 0.25 milliard ⑬ 480×10^{12}

▶ **Write each of the following in standard form:**

⑭ 5.4×10^3 ⑮ 4.32×10^{-2} ⑯ 5.37×10^5

▶ **Write the result of each of the following in scientific notation:**

⑰ $(4.5 \times 10^7) \times (4 \times 10^8)$ ⑱ $(4.8 \times 10^{-7}) \div (0.8 \times 10^5)$

⑲ $(9.8 \times 10^{-5}) + (4.9 \times 10^{-6})$ ⑳ $(2.4 \times 10^5) - (4.2 \times 10^4)$

▶ **Arrange each of the following in an ascending order:**

㉑ 73×10^{11} , 6.9×10^{12} , 0.537×10^{13} ㉒ 4,300,000 , 3.4×10^6 , 0.37×10^7

Third **Analysis and Subjects Integration**

㉓ **Astronomy:** The temperature at the center of the sun is approximately 15 million Celsius. Write this temperature in scientific notation.

㉔ **Scientists:** Dr. Ahmed Zewail invented an ultra-fast camera that uses laser beams to capture chemical reactions occurring at an extremely rapid pace, measured in femtoseconds, which is a unit equivalent to one part of million milliard parts of a second. Express this unit in scientific notation.

With knowledge, nations progress

㉕ **Science and Astronomy:** If the mass is measured in grams, write each of the following in scientific notation:

① Mass of a hydrogen atom: 0.00000000000000000000000167

② Mass of the moon: 73,600,000,000,000,000,000,000,000

㉖ **Science:** The flapping speed of a honeybee's wings is approximately 230 times per second. Write in scientific notation the number of wings flaps in 10 minutes.

㉗ **Geography:** Arrange the areas of the following oceans in a descending order:

Ocean	Indian	Pacific	Southern	Arctic	Atlantic
Area (in square kilometers)	7.3×10^7	1.69×10^8	2.1×10^7	1.4×10^7	8.6×10^7

Creative Thinking

㉘ Write five numbers in scientific notation that lie between 10,000 and 20,000.

Square Roots and Cube Roots

Learning Outcomes

- Learn the concept of the square root.
- Learn the concept of the cube root.
- Find the square root and cube root of numbers.

Vocabulary

- Square root
- Cube root
- Perfect square
- Perfect cube

Note that

- Squares of integers:
  $1^2, 2^2, 3^2, 4^2, 5^2, ...$
  Equal:
  1, 4, 9, 16, 25, ...
  These are perfect squares.

- Cubes of integers:
  $1^3, 2^3, 3^3, 4^3, 5^3, ...$
  Equal:
  1, 8, 27, 64, 125, ...
  These are perfect cubes.

Critical Thinking

In the set of integers, why is there no square root for a negative number?

Get Ready!

The Great Pyramid or Pyramid of Khufu is the only remaining wonder of the Seven Wonders of the Ancient World, located in the Giza pyramid area in Egypt, which is listed as a UNESCO World Heritage Site. The current height of the pyramid is approximately 139 meters, and its base is a square with an area of approximately 52,900 square meters.

The Great Pyramid (Pyramid of Khufu)

Can you find the side length of the base of the pyramid?

In this lesson, you will learn how to find square roots and cube roots, which will enable you to solve such problems.

Think & Discuss!

Consider the following pattern and then find the number of small squares in shape (7), and if you know that the number of small squares in a certain shape equals 144, what is the number of that shape?

Shape (1) Shape (2) Shape (3) Shape (4) Shape (5)
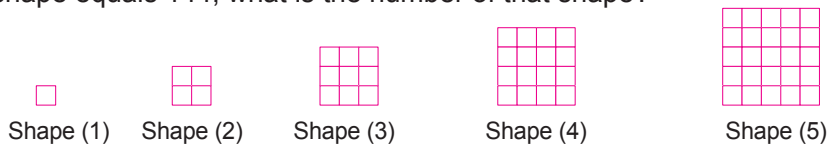
Consider the following pattern and then find the number of small cubes in shape (6), and if you know that the number of small cubes in a certain shape equals 729, what is the number of that shape?

Shape (1) Shape (2) Shape (3) Shape (4) Shape (5)
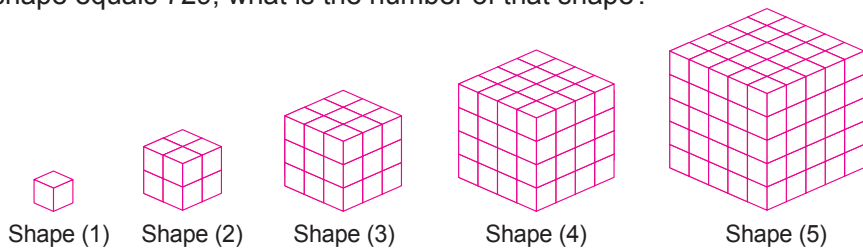
Learn!

The square root of a perfect square number

The square root of a perfect square number (*a*) is the number whose square equals (*a*).

The perfect square number has two square roots, one is positive and the other is negative, each of them is the additive inverse of the other.

For example: The number 25 has two square roots, which are:
$$5, -5 \quad \text{because: } (5)^2 = 25, \quad (-5)^2 = 25$$

The symbol « $\sqrt{}$ » denotes the positive square root of a number.

For example: • The positive square root of the number 4 is 2,
and it is written as: $\sqrt{4} = 2$

• The negative square root of the number 9 is -3,
and it is written as: $-\sqrt{9} = -3$

• The square roots of the number 64 are 8, -8,
and they are written as: $\pm\sqrt{64} = \pm 8$

■ Note that

1. $\sqrt{0} = 0$

2. $\sqrt{a^2} = |a|$

For example:

$\sqrt{(-7)^2} = |-7| = 7$

3. $\sqrt{a^{2n}} = |a^n|$

where n is an integer
For example:

$\sqrt{a^6} = |a^3|$

$\sqrt{a^4} = |a^2| = a^2$

Example 1

Find each of the following in its simplest form:

1. $\sqrt{196}$

2. $-\sqrt{\dfrac{9}{16}}$

3. $\pm\sqrt{0.64}$

4. $\sqrt{6\dfrac{1}{4}}$

5. $\sqrt{\left(\dfrac{-3}{5}\right)^2}$

6. $\sqrt{100 - 36}$

✅

1. $196 = \underbrace{2 \times 2}_{} \times \underbrace{7 \times 7}_{}$

$\sqrt{196} = 2 \times 7 = 14$

196	2
98	2
49	7
7	7
1	

2. $-\sqrt{\dfrac{9}{16}} = -\dfrac{3}{4}$

3. $\pm\sqrt{0.64} = \pm\sqrt{\dfrac{64}{100}} = \pm\dfrac{8}{10} = \pm\dfrac{4}{5}$

4. $\sqrt{6\dfrac{1}{4}} = \sqrt{\dfrac{25}{4}} = \dfrac{5}{2} = 2\dfrac{1}{2}$

■ Technology

You may use a calculator to verify your answers

5. $\sqrt{\left(\dfrac{-3}{5}\right)^2} = \left|\dfrac{-3}{5}\right| = \dfrac{3}{5}$

6. $\sqrt{100 - 36} = \sqrt{64} = 8$

■ Self-Evaluation ①

Find each of the following:

1. $\pm\sqrt{576}$

2. $\sqrt{\left(\dfrac{-5}{8}\right)^2}$

3. $-\sqrt{11\dfrac{1}{9}}$

4. $\sqrt{15^2 - 9^2}$

Solving equations using the square root

If $x^2 = a$ where $a \geq 0$, then $x = \pm\sqrt{a}$

For example: If $x^2 = 36$, then $x = \pm\sqrt{36} = \pm 6$

■ Self-Evaluation ②

Find the value of x in each of the following:

1. $x^2 - 1 = 8$

2. $3x^2 - 5 = 43$

Example 2

Find the value of x in each of the following:

1. $x^2 + 1 = 65$

2. $2x^2 - 5 = 13$

✅

1. $\because x^2 + 1 = 65$

$\therefore x^2 = 65 - 1 = 64$

$\therefore x = \pm\sqrt{64} = \pm 8$

2. $\because 2x^2 - 5 = 13$

$\therefore 2x^2 = 13 + 5 = 18$

$\therefore x^2 = \dfrac{18}{2} = 9$

$\therefore x = \pm\sqrt{9} = \pm 3$

■ **Self-Evaluation** ③

A farmer owns a square plot of land has an area of 1024 square meters.
He decided to divide it into 4 equal square plots.
What is the length of each side of the four plots?

Example ③

Link to Life: A square garden has an area of 400 square meters, and a fence is to be constructed around it. What is the length of the garden's fence?

✅

Let the length of the side of the garden be ℓ meters, then its area is ℓ^2 square meters.

$\therefore \ell^2 = 400$ $\qquad\qquad \therefore \ell = \sqrt{400} = 20$

Thus, the length of the side of the garden is 20 meters.

■ **Think**

In the solution of the adjacent example, why is $\ell \neq -20$?

Since the length of the garden's fence (P) is the perimeter of the garden,

$\therefore P = 4 \times 20 = 80$

Thus, the length of the garden's fence is 80 meters.

■ **Note that**

① $\sqrt[3]{0} = 0$

② $\sqrt[3]{a^{3n}} = a^n$

where n is an integer.

For example:

$\sqrt[3]{x^3} = x$

$\sqrt[3]{x^{27}} = x^9$

The cube root of a perfect cube number

The cube root of a perfect cube number (a) is the number whose cube equals (a).

The cube root of a perfect cube number has the same sign as this number.

For example: • The number (8) has a cube root of (2) because: $2^3 = 8$
  • The number (-8) has a cube root of (-2)
     because: $(-2)^3 = -8$

The symbol « $\sqrt[3]{}$ » denotes the cube root of a number.

For example: The cube root of the number 125 is 5, written as: $\sqrt[3]{125} = 5$

■ **Self-Evaluation** ④

Find each of the following:

① $\sqrt[3]{512}$ ② $\sqrt[3]{-125}$

③ $\sqrt[3]{0.027}$ ④ $\sqrt[3]{3\frac{3}{8}}$

⑤ $\sqrt[3]{(-9)^3}$ ⑥ $\sqrt[3]{x^9}$

Example ④

Find each of the following in its simplest form:

① $\sqrt[3]{216}$ ② $\sqrt[3]{-\dfrac{27}{125}}$ ③ $\sqrt[3]{0.343}$

④ $\sqrt[3]{15\dfrac{5}{8}}$ ⑤ $\sqrt[3]{(-5)^3}$ ⑥ $\sqrt[3]{y^{12}}$

✅

① $216 = \underbrace{2 \times 2 \times 2} \times \underbrace{3 \times 3 \times 3}$

$\sqrt[3]{216} = \qquad 2 \quad \times \quad 3 \quad = 6$

② $\sqrt[3]{-\dfrac{27}{125}} = -\dfrac{3}{5}$

216	2
108	2
54	2
27	3
9	3
3	3
1	

☑

③ $\sqrt[3]{0.343} = \sqrt[3]{\dfrac{343}{1000}} = \dfrac{7}{10}$

④ $\sqrt[3]{15\dfrac{5}{8}} = \sqrt[3]{\dfrac{125}{8}} = \dfrac{5}{2} = 2\dfrac{1}{2}$

⑤ $\sqrt[3]{(-5)^3} = -5$ ⑥ $\sqrt[3]{y^{12}} = y^4$

Example 5

Simplify to its simplest form: $\sqrt[3]{\dfrac{729}{64}} \times \sqrt{\dfrac{64}{9}} \times \left(\dfrac{-8}{3}\right)^0$

☑

$\sqrt[3]{\dfrac{729}{64}} \times \sqrt{\dfrac{64}{9}} \times \left(\dfrac{-8}{3}\right)^0 = \dfrac{9}{4} \times \dfrac{8}{3} \times 1 = 6$

> **Solving equations using the cube root**

If $x^3 = a$, then $x = \sqrt[3]{a}$

For example: If $x^3 = 1000$, then $x = \sqrt[3]{1000} = 10$

Example 6

If $2x^3 + 3 = 253$, what is the value of x?

☑

$\because 2x^3 + 3 = 253$ $\therefore 2x^3 = 253 - 3 = 250$

$\therefore x^3 = \dfrac{250}{2} = 125$ $\therefore x = \sqrt[3]{125} = 5$

Example 7

Link to Life: A piece of clay with a volume of 512 cube centimeters is shaped into a cube.
What is the lateral and total (surface) area of the cube?

☑

Assume the length of the edge of the cube is s, then its volume is s^3

$\therefore s^3 = 512$ $\therefore s = \sqrt[3]{512} = 8$

i.e The edge length of the cube is 8 $cm.$

$\therefore$ The area of one face $= s^2 = 8^2$
$\qquad\qquad\qquad\qquad = 64$ square centimeters

The lateral area $= 4\,s^2 = 4 \times 64$
$\qquad\qquad\qquad = 256$ square centimeters

The total (surface) area $= 6\,s^2 = 6 \times 64$
$\qquad\qquad\qquad\qquad = 384$ square centimeters
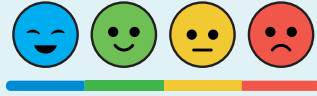
Lesson Assessment

▶ **Choose the correct answer from the given ones:**

(1) If $\sqrt{x} = 5$, what is the value of x?

  (a) 10   (b) 20   (c) 25   (d) $\pm\,25$

(2) If $x^3 = -27$, what is the value of x?

  (a) -3 (b) 3   (c) $\pm\,3$ (d) -9

(3) If $a = 5^3$, what is the value of $\sqrt[3]{a}$?

  (a) 3   (b) 5   (c) 25   (d) 125

(4) If $\sqrt[3]{b} = -8$, what is the value of b?

  (a) 2   (b) -2 (c) 64   (d) -512

(5) What is the value of $\sqrt{(-5)^2}$?

  (a) -5 (b) 5   (c) $\pm\,5$ (d) 25

(6) If a and b are the two square roots of the number c, what is $a + b$ equal to?

  (a) $2\,a$ (b) $2\,b$ (c) 1   (d) 0

▶ **Find the value of each of the following:**

(7) $\sqrt{16}$

(8) $-\sqrt{100}$

(9) $\pm\sqrt{2,500}$

(10) $\sqrt{\left(\dfrac{-4}{9}\right)^2}$

(11) $\sqrt{0.81}$

(12) $\sqrt{36 + 64}$

(13) $\sqrt{\dfrac{4\,a^2}{9\,b^4}}$

(14) $\sqrt{2^2 + 3^2 + 6^2}$

▶ **Find the value of each of the following:**

(15) $\sqrt[3]{64}$

(16) $\sqrt[3]{-1}$

(17) $\sqrt[3]{\dfrac{-64}{27}}$

(18) $\sqrt[3]{64 \times 2^3}$

(19) $\sqrt[3]{343 - 7^3}$

(20) $\sqrt{4} - \sqrt[3]{-8}$

(21) $\sqrt[3]{\dfrac{8\,x^3}{125\,y^6}}$

(22) $\sqrt[3]{0.125} + \sqrt{12\dfrac{1}{4}}$

▶ **Find the solution set for each of the following equations in Z:**

(23) $x^2 = 25$

(24) $x^3 = 64$

(25) $x^3 + 26 = -1$

(26) $2x^2 + 1 = 33$

(27) $3x^3 - 4 = 2x^3 + 4$

(28) $(x - 1)^3 + 2 = -6$

▶ **Simplify each of the following:**

(29) $\sqrt{\dfrac{9}{4}} + \sqrt[3]{\dfrac{-27}{8}} + \left(\dfrac{4}{9}\right)^0$

(30) $\left(\dfrac{3}{2}\right)^2 + \sqrt{\dfrac{25}{4}} + \sqrt[3]{\dfrac{125}{64}}$

▶ **Choose the correct answer from the given ones:**

㉛ What is the multiplicative inverse of the number $\sqrt{\dfrac{9}{25}}$ in its simplest form?

(a) $-\dfrac{3}{5}$　(b) $\dfrac{3}{5}$　(c) $-\dfrac{5}{3}$　(d) $\dfrac{5}{3}$

㉜ What is the value of $\sqrt[3]{\sqrt{64}}$?

(a) 2　(b) 4　(c) 8　(d) 64

㉝ If $x = \sqrt{\dfrac{1}{9}}$, what is the value of x^3?

(a) $\dfrac{1}{3}$　(b) $\dfrac{1}{9}$　(c) $\dfrac{1}{27}$　(d) $\dfrac{1}{81}$

㉞ If $-\sqrt{25} = \sqrt[3]{y}$, what is the value of y?

(a) -5　(b) 5　(c) -125　(d) 125

| Third | **Analysis and Subjects Integration** |

㉟ **Arts:** Fatma painted a square-shaped picture with an area of a^4b^2 square centimeters. If she wants to make a frame around it, what is the length of the frame if $a = 5$, $b = 4$?

㊱ **Solids:** A cube has a volume of x^6, find the sum of the lengths of its edges if $x = 10$.

Arts in all their forms contribute to the development of creativity, thinking, and imagination skills.

㊲ **Geometry:** In the opposite figure, if the area of each square is 36 square centimeters, what is the perimeter of the shape?

㊳ **Solids:** The total (surface) area of a cube is 294 square centimeters. Find the length of its edge.

㊴ **Spot the mistake:**

Iman wrote the opposite two sentences while doing her homework.
Correct the mistake that Iman made.

$$\sqrt[3]{(-4)^3} = |-4| = 4$$

$$\sqrt{3^2 + 4^2} = 3 + 4 = 7$$
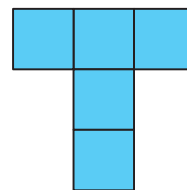
Creative Thinking

㊵ In the opposite figure,

A, B, C are three squares.
The area of square C is 9 square units,
the area of square B is 36 square units;
find the area of square A.
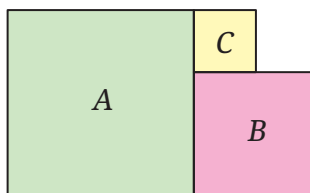
Evaluate your *understanding!*

How well do you understand square roots and cube roots? Tick the right box.
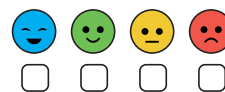
▶ **Choose the correct answer from the given ones:**

(1) If the world's population is approximately 8 milliard, what is the scientific notation for the world population?

(a) 8×10^{10} (b) 8×10^{9}

(c) 80×10^{8} (d) 8×10^{8}

(2) Which of the following equals $2x^{-1}$?

(a) $2x$ (b) $\frac{1}{2}x$

(c) $\frac{1}{2x}$ (d) $\frac{2}{x}$

(3) Which of the following equals $\sqrt[3]{(-8)^2}$?

(a) -4 (b) -2 (c) 2 (d) 4

(4) Which of the following equals $\sqrt{9x^2}$?

(a) $3x$ (b) $9x$ (c) $3x^2$ (d) $3|x|$

▶ **Complete each of the following with the correct answer:**

(5) If $0.000046 = 4.6 \times 10^n$, then $n = $ ············

(6) $\sqrt{16} - \sqrt[3]{-27} = $ ············

(7) If $x^3 + 4 = 12$, then $x = $ ············

(8) If $3^4 + 3^4 + 3^4 = 3^n$, then $n = $ ············

▶ **Answer the following questions:**

(9) The area of a square equals the area of a triangle with a base length of 9 cm and a height corresponding to this base of 8 cm. Find the length of the square's side.

(10) Simplify to its simplest form: $\dfrac{a^7 \times a^8 \times a^2}{a^3 \times a^9 \times a^5}$ where $a \neq 0$

(11) Simplify to its simplest form: $\sqrt{\dfrac{81}{49}} + \left(\dfrac{3}{4}\right)^0 + \sqrt[3]{\dfrac{125}{343}}$

(12) Write the result of the following in scientific notation: $(5.2 \times 10^9) - (8.5 \times 10^8)$

Unit One Activity — Planets of the Solar System

▶ **Activity Aim:** To use artificial intelligence technology to assist in formulating a problem related to the concept of scientific notation of numbers and solving it.

▶ **Implementation Steps: In collaboration with a friend, attempt to do the following:**

1 Use an artificial intelligence application to assist in formulating a mathematical problem related to one of the following pieces of information (speed of light - mass of the planet - distance of the planet from the sun).

2 Rephrase the problem in your own words.

3 Collect the data related to the problem and organize it in a table to clarify the information pertaining to each planet in the solar system.

4 Outline the steps to solve the problem, emphasising how to benefit from the scientific notation of numbers.
Example of problem formulation:
If you know that ...... then find ......
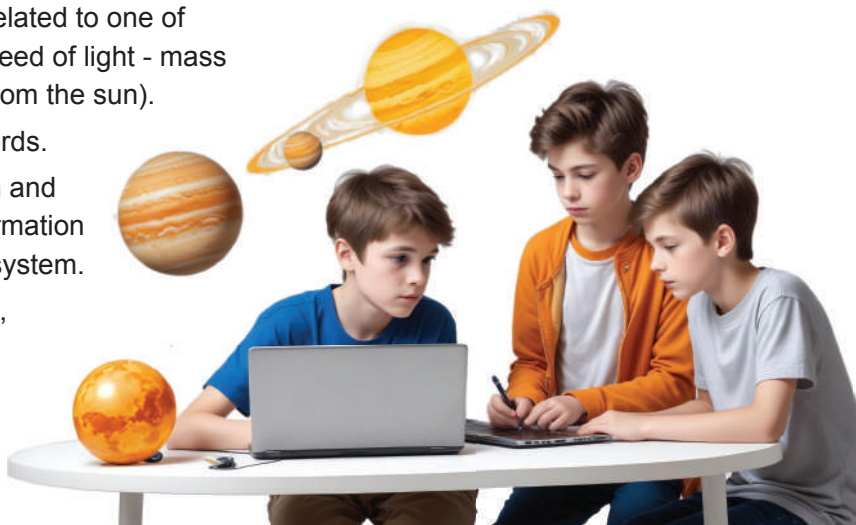
UNIT 2

Algebra
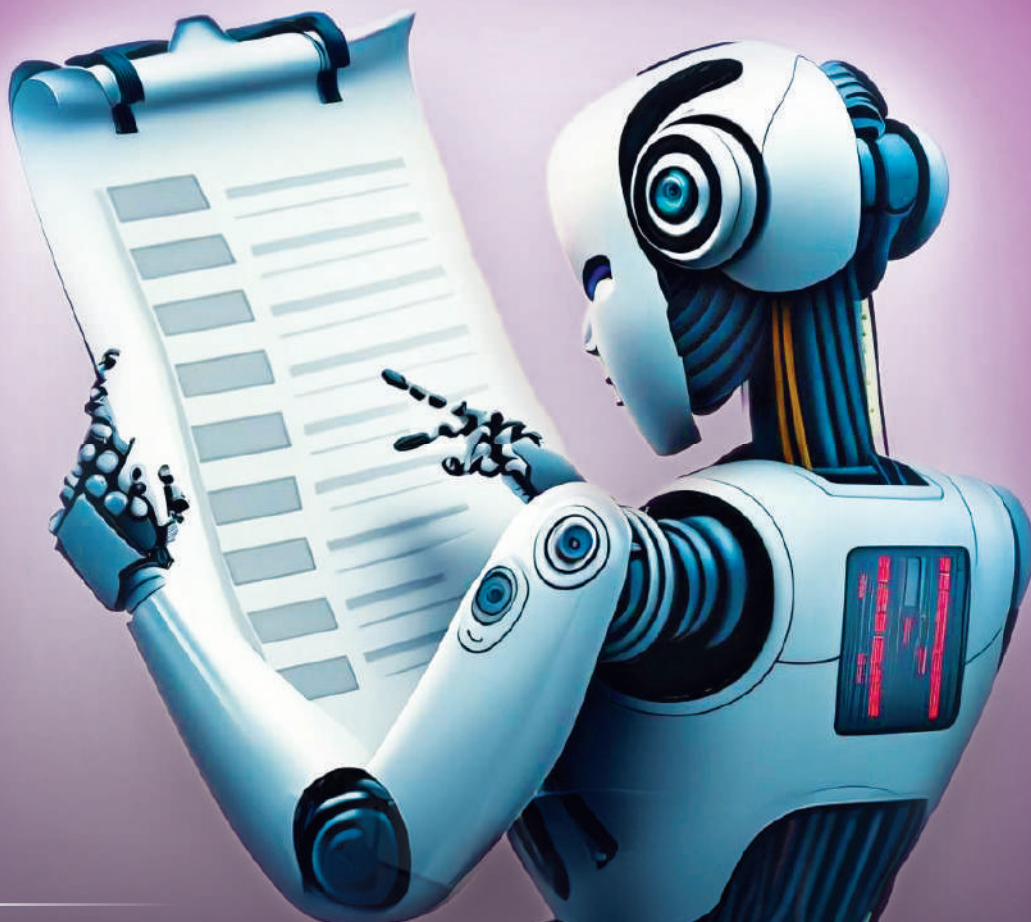
Artificial Intelligence (AI) is used to provide detailed solutions to some complex algebraic problems.

▶ **Can artificial intelligence applications be used to perform algebraic operations involving equations and inequalities?**

Issues and Life Skills
- Mathematical Understanding
- Information Technology
- Creative Thinking
- Logical Thinking
- Problem Solving

Values
- Cooperation
- Responsibility
- Appreciation of Beauty
- Respecting Others
- Risk Avoidance
- Work

Inequalities

Learning Outcomes

- Learn a first degree inequality in one variable.
- Form a first degree inequality in one variable.
- Solve a first degree inequality in one variable using the properties of inequality.

Vocabulary

- Inequality
- Less Than
- Greater Than
- Less Than or Equal to
- Greater Than or Equal to

Notation in Mathematics

The symbols $\geq$, $\leq$ combine the symbols $>$, $<$ with the equality sign ($=$).

Note that

The inequality $x < 5$ is the same as the inequality $5 > x$.

Get Ready!

The Egypt Stadium in the New Administrative Capital is one of the largest stadiums in the Middle East and Africa in terms of spectator capacity, with an approximate capacity of about 90,000 spectators. What is the inequality that represents the number of spectators (x) expected to attend one of our national team's matches if 1,000 invitations are sent to VIPs?

In this lesson, you will learn to form and solve a first-degree inequality in one variable, which can help you solve such real-life problems.

Egypt Stadium in the New Administrative Capital

Think & Discuss!

Osama reduced his car speed by 15 km/h upon reaching a residential area where the maximum speed limit is 20 km/h and he did not exceed the speed limit. Which of the following expresses the speed of the car (x) upon reaching the residential area?

$$x - 15 \leq 20 \qquad x + 15 \leq 20 \qquad x - 15 > 20$$

Learn!

The Concept of Inequality

An inequality consists of two mathematical expressions, with one of the inequality signs ($>$, $<$, $\geq$, $\leq$) between them.

Examples:

- $x < 5$ "It is read as x is less than 5"
- $x > -2$ "It is read as x is greater than -2"
- $x \leq -3$ "It is read as x is less than or equal to -3"
- $x \geq 100$ "It is read as x is greater than or equal to 100"

Other Examples:

- $2x - 1 > -1$
- $5x + 1 \leq 16$
- $3x + y > 3$

Our study in this lesson will focus on first degree inequalities in one variable.

A first degree inequality in one variable

It is an inequality that contains only one variable raised to the exponent one.

For example: $2x - 1 \leq 6$, $x - 3 > 3x - 1$, $2(2x - 1) > 6$

■ Self-Evaluation ①

Express each of the following situations with an appropriate inequality:

❶ Your height must exceed 110 *cm* to play one of the games at an amusement park.

❷ The maximum speed of your car is 80 *km/h*.

❸ You should be at least 12 years old to use a mobile phone.

Writing the Inequality

Example 1

Express each of the following situations with an appropriate inequality:

❶ You must be at least 18 years old to obtain a driving licence.

❷ The hall accommodates a maximum of 120 people.

❸ If 2 is subtracted from three times a number, the result is greater than 7.

✔️

① Let the age be x
$$\therefore x \geq 18$$

② Let the number of people be y
$$\therefore y \leq 120$$

③ Let the number be x, then its three times is $3x$
$$\therefore 3x - 2 > 7$$

Solving the Inequality

- Solving the inequality means to find the values of the variable that satisfy the inequality.
- **Substitution Set :** It is the set to which all possible values of the variable in the inequality belong.
- **Solution Set :** It is a subset of the substitution set, and its elements satisfy the inequality.

Solving the Inequality Using Properties of inequality

When solving a first degree inequality in one variable, use the same method as in solving first degree equations in one variable, taking into account the properties of inequality.

Properties of Inequality

If A, B, and C are three numbers, then these numbers have the following properties:

■ **Property** ❶

When the same number is added to both sides of the inequality, it remains true.

❶ **Addition Property :**

If $A > B$ then $A + C > B + C$

For example: $\boxed{5 > 3}$ then $5 + 1 > 3 + 1$ hence $\boxed{6 > 4}$ ✔

■ **Property** ❷

When the same number is subtracted from both sides of the inequality, it remains true.

❷ **Subtraction Property :**

If $A > B$ then $A - C > B - C$

For example: $\boxed{5 > 3}$ then $5 - 2 > 3 - 2$ hence $\boxed{3 > 1}$ ✔

■ **Property** ❸

- When both sides of the inequality are multiplied by a positive number, it remains true.
- When both sides of the inequality are multiplied by a negative number, the direction of the inequality changes in order for the inequality to remain true.

❸ **Multiplication Property :**

If $A > B$ then
$$A \times C > B \times C \quad \text{if } C > 0$$
$$A \times C < B \times C \quad \text{if } C < 0$$

For example: $\boxed{4 > 2}$

then
$$4 \times 5 > 2 \times 5 \quad \text{hence } \boxed{20 > 10} ✔$$
$$4 \times (-5) < 2 \times (-5) \quad \text{hence } \boxed{-20 < -10} ✔$$

• When both sides of the inequality are divided by a positive number, it remains true.
• When both sides of the inequality are divided by a negative number, the direction of the inequality changes in order for the inequality to remain true.

■ **Critical Thinking**

If $a \neq 0$, $b \neq 0$ and if $a < b$, which is greater, $\frac{1}{a}$ or $\frac{1}{b}$?

■ **Self-Evaluation** ②

Find the solution set for each of the following inequalities:

$$x + 2 > -1$$
$$2x \geq 10$$
$$4x - 1 < 11$$

If the substitution set is:
① N　② Z　③ Q

■ **Note that**

The solution set of the inequality in Q is expressed in the form of a set builder.

■ **Self-Evaluation** ③

Find the solution set in Z for each of the following inequalities:

① $7 - 3x > -5$

② $2(x + 5) - 7 > 9$

③ $4(x + 3) > 7x - 9$

---

④ **Division Property :**

If $A > B$　　then
$$\frac{A}{C} > \frac{B}{C} \quad \text{if } C > 0$$
$$\frac{A}{C} < \frac{B}{C} \quad \text{if } C < 0$$

For example : $\boxed{4 > 2}$　then
$$\frac{4}{2} > \frac{2}{2} \quad \text{hence } \boxed{2 > 1} \checkmark$$
$$\frac{4}{-2} < \frac{2}{-2} \quad \text{hence } \boxed{-2 < -1} \checkmark$$

Example ②

Find the solution set for the inequality: $2x + 5 \leq 11$

If the substitution set is: ① N　　② Z　　③ Q

✔

∵ $2x + 5 \leq 11$

∴ $2x + 5 - 5 \leq 11 - 5$

(Subtracting 5 from both sides)

∴ $2x \leq 6$

∴ $\dfrac{2x}{2} \leq \dfrac{6}{2}$ 　(Dividing both sides by 2)

∴ $x \leq 3$

① The solution set in $N = \{0, 1, 2, 3\}$

② The solution set in $Z = \{3, 2, 1, 0, -1, -2, ...\}$

③ The solution set in $Q = \{a : a \in Q, a \leq 3\}$

■ **Diversified Strategies**

The solution can be done by reversing the operation

∵ $2x + 5 \leq 11$

∴ $2x \leq 11 - 5$

∴ ② $x \leq 6$

∴ $x \leq \dfrac{6}{2}$

∴ $x \leq 3$

Example ③

Find the solution set in Z for each of the following inequalities:

① $1 - 2x < 5$　　② $3(2x - 1) > 9$　　③ $2(2x + 3) \leq 5x + 2$

✔

① ∵ $1 - 2x < 5$

∴ $1 - 2x - 1 < 5 - 1$　　　　(Subtracting 1 from both sides)

∴ $-2x < 4$

∴ $\dfrac{-2x}{-2} > \dfrac{4}{-2}$　　　　(Dividing both sides by -2)

∴ $x > -2$

∴ The solution set $= \{-1, 0, 1, 2, ...\}$

■ **Verifying the Solution**

You can verify the solution of problem ❷ as follows:

Write the original inequality $3(2X-1)>9$

Substitute X with 3

$3(2\times 3-1)\overset{?}{>}9$

$3(6-1)\overset{?}{>}9$

$3\times 5\overset{?}{>}9$

The statement is true:

$15>9$ ✓

Thus, 3 is a solution to the inequality.

Repeat the same steps by substituting X with 4, and substituting X with 5 to ensure that they are solutions to the inequality.

▸ Can you verify the solution to problem ❸?

② $\because 3(2X-1)>9$

$\therefore 6X-3>9$ (Distributive Property)

$\therefore 6X-3+3>9+3$ (Adding 3 to both sides)

$\therefore 6X>12$

$\therefore \dfrac{6X}{6}>\dfrac{12}{6}$ (Dividing both sides by 6)

$\therefore X>2$

$\therefore$ The solution set $=\{\,3,4,5,...\,\}$

③ $\because 2(2X+3)\le 5X+2$

$\therefore 4X+6\le 5X+2$ (Distributive Property)

$\therefore 4X-5X\le 2-6$ (Combining like terms)

$\therefore -X\le -4$

$\therefore -X\times(-1)\ge -4\times(-1)$ (Multiplying both sides by -1)

$\therefore X\ge 4$

$\therefore$ The solution set $=\{\,4,5,6,7,...\,\}$

■ **Diversified Strategies**

Problem ❷ can be solved by dividing both sides by 3 without using the distributive property.

$\because \dfrac{3(2X-1)}{3}>\dfrac{9}{3}$

$\therefore 2X-1>3$

$\therefore 2X-1+1>3+1$

$\therefore 2X>4$

$\therefore \dfrac{2X}{2}>\dfrac{4}{2}$

$\therefore X>2$

■ **Self-Evaluation** ④

The maximum number of people an elevator can carry is 4, such that their total mass does not exceed 300 *kg*. If there are 3 people in the elevator with a total mass of 225 *kg*,

write an inequality that expresses the mass X *kg* of the fourth person who can enter the elevator without violating the safety guidelines, and solve the inequality to find the maximum value of X.

Example ④

Link to Life: If a teacher wishes to purchase 5 pens of the same type to distribute to his outstanding students in the class, ensuring that he does not spend more than 150 pounds, including 20 pounds for shipping costs, write an inequality that expresses the price of one pen, and solve the inequality to find the maximum price for one pen.

✅

Assuming the price of one pen is X pounds.

$\therefore 5X+20\le 150$

$\therefore 5X\le 150-20$

$\therefore 5X\le 130$

$\therefore X\le \dfrac{130}{5}$

$\therefore X\le 26$

Thus, the maximum price for one pen is 26 pounds.

First — Measuring Conceptual Understanding

▶ **Choose the correct answer from the given ones:**

1 What is the inequality that expresses that the temperature x is less than 40°?

(a) $x < 40°$ (b) $x > 40°$

(c) $x \leq 40°$ (d) $x \geq 40°$

2 Which of the following inequalities expresses the following situation: "Omar needs at least two hours to complete the homework"?

(a) $x < 2$ (b) $x \leq 2$

(c) $x > 2$ (d) $x \geq 2$

3 What is the inequality that expresses that twice the number x is less than 5 ?

(a) $x + 2 < 5$ (b) $x - 2 < 5$

(c) $2x < 5$ (d) $2x > 5$

4 If $x - 1 > 4$, which of the following could be the value of x?

(a) 3   (b) 4

(c) 5   (d) 7

5 Which of the following inequalities has $x = -7$ as one of its solutions in Z?

(a) $x > -7$ (b) $x < -7$

(c) $x > -6$ (d) $-x \geq -7$

6 Which of the following inequalities has $x = -4$ as one of its solutions in Q?

(a) $x - 2 \geq -4$ (b) $2x > -8$

(c) $x + 2 > -3$ (d) $-x > 4$

▶ **Spot the mistake:**

7 Both Mustafa and Salma solved the inequality $2 - 3x < 17$ in Z.

Which of them followed the correct method to solve the inequality? Discuss.

Mustafa's Solution:

$\because 2 - 3x < 17$

$\therefore -3x < 17 - 2$

$\therefore -3x < 15$

$\therefore \dfrac{-3x}{-3} > \dfrac{15}{-3}$

$\therefore x > -5$

$\therefore$ The Solution Set = $\{-4, -3, -2, …\}$

Salma's Solution:

$\because 2 - 3x < 17$

$\therefore -3x < 17 - 2$

$\therefore -3x < 15$

$\therefore \dfrac{-3x}{-3} < \dfrac{15}{-3}$

$\therefore x < -5$

$\therefore$ The Solution Set = $\{-6, -7, -8, …\}$

Second — Applying Scientific Concepts

▶ **Express each of the following situations with an appropriate inequality:**

8 To obtain the discount, your purchases must exceed 500 pounds.

9 An accountant with at least 3 years of experience is required.

10 The mass of the suitcase must not exceed 7 kilograms in order to carry it inside the aircraft cabin.

▶ **Find the solution set for each of the following inequalities in N :**

(11) $X - 2 > 1$ (12) $-2X \leq 0$ (13) $2X - 3 < 7$ (14) $\frac{1}{3}X + 3 \leq 1$

▶ **Find the solution set for each of the following inequalities in Z :**

(15) $5 - 3X \geq 14$ (16) $2(3 - 2X) < 4$ (17) $3X + 7 < 7X + 3$

▶ **Find the solution set for each of the following inequalities in Q :**

(18) $2(X + 5) - 3 < 12$ (19) $X - 2 \leq 3X + 7$ (20) $3(X - 7) \geq 7(X - 3)$

Third   **Analysis and Subjects Integration**

(21) **Sports:** In a basketball match, Marwan scored at least 12 points more than Rami. If Rami scored 15 points, write an appropriate inequality to express this situation and solve it, then find the minimum number of points that Marwan scored.

(22) **Savings:** Hamza needs to save at least 250 pounds to buy a new toy. If he already has 100 pounds and he can save 20 pounds each week from his allowance, write an appropriate inequality to express this situation and solve it, then find the minimum number of weeks Hamza will need to save the money to buy the toy.

> Exercising helps you achieve your goals.

> Your support brings a smile to the faces of those in need.

(23) **Shopping:** Aliya wants to buy some clothes for a charity. If the price of one shirt is 240 pounds, and she has a discount card for 400 pounds and a budget not exceeding 3,200 pounds, write an appropriate inequality to express this situation and solve it, then find the maximum number of shirts she can buy of the same type.

Creative Thinking

(24) Find all the integer values of X that satisfy the two inequalities $5X - 2 > 3$, $7 - X \geq 3$ simultaneously.

(25) Find the smallest three consecutive even numbers whose sum is greater than 96.

(26) **Geometry:** A triangle has side lengths of 6 cm, 8 cm, and $(2X - 4)$ cm. What are the possible integer values of X?

> **Evaluate** your understanding!
>
> How well do you understand inequalities?
> Tick the right box

Multiplying an Algebraic Term by an Algebraic Term or an Algebraic Expression

Get Ready!

A tennis court, its length is two meters more than twice its width. The ground is to be covered with artificial grass. Write an algebraic expression to represent the area of the court, and if the width of the court is 11 meters, what is the numerical value of its area?

In this lesson, you will learn how to multiply algebraic terms and how to multiply an algebraic term by an algebraic expression, which can help you solve such problems.

Think & Discuss!

Nader cut a piece of paper in the shape of the letter T. Its dimensions are shown in the opposite figure.

Can you write an algebraic expression that represents the area of this piece of paper?

$2x+2$

x

$3x$

x
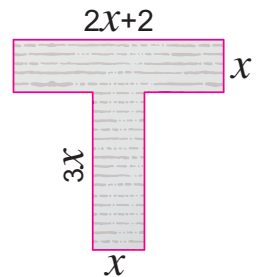
Learn!

Multiplying an Algebraic Term by Another Algebraic Term

When multiplying an algebraic term by another, multiply the coefficients and add the exponents of the variables that have the same base.

$$a\,x^m \times b\,x^n = a \times b\,x^{m+n}$$

Add the exponents

For example: $4\,x^2 \times 6\,x^3 = (4 \times 6)\,x^{2+3} = 24\,x^5$

Multiply the coefficients

Example 1

Find the result of each of the following:

❶ $(-4\,a^3)(3\,a^5)$ ❷ $(-3\,a^3\,b^2)(-2\,a\,b^4)$ ❸ $(9\,x^3\,y)(-2\,x^2\,y\,z^5)$

✓

① $(-4\,a^3)(3\,a^5) = (-4 \times 3) \times a^{3+5} = -12\,a^8$

② $(-3\,a^3\,b^2)(-2\,a\,b^4) = (-3 \times (-2)) \times a^{3+1} \times b^{2+4} = 6\,a^4\,b^6$

③ $(9\,x^3\,y)(-2\,x^2\,y\,z^5) = (9 \times (-2)) \times x^{3+2} \times y^{1+1} \times z^5$

$= -18\,x^5 y^2 z^5$

■ Modelling

The product of $a\,(b+c)$ can be found geometrically using the area model of a rectangle as follows:

$a\,(b+c)=ab+ac$

Multiplying an Algebraic Term by an Algebraic Expression Consisting of Two or More Terms

When multiplying an algebraic term by an algebraic expression consisting of two or more terms, use the distributive property.

$$a\,(b+c)=ab+ac \qquad a\,(b-c)=ab-ac$$

Examples: • $2X\,(X+5)=(2X)\,(X)+(2X)\,(5)=2X^2+10X$

• $5X\,(X-3)=(5X)\,(X)-(5X)\,(3)=5X^2-15X$

┌── **The vertical method** ──

You can use the vertical method to find the product as shown in the opposite figure.

$$X+5$$
$$\times\ 2X$$
$$\overline{2X^2+10X}$$

■ Self-Evaluation ②

Find the result of each of the following:

① $-3X\,(X-5)$

② $2X^2\,(4X^2-5X-7)$

③ $-3a^2b\,(2ab^2-2b)$

Example ②

Find the result of each of the following:

① $-4a\,(3a^2-2a+1)$

② $2Xy\,(4X^2+3Xy^2-5y)$

✓

① $-4a\,(3a^2-2a+1)=(-4a)\,(3a^2)-(-4a)\,(2a)+(-4a)\,(1)$
$$=-12a^3-(-8a^2)+(-4a)$$
$$=-12a^3+8a^2-4a$$

② $2Xy\,(4X^2+3Xy^2-5y)=(2Xy)\,(4X^2)+(2Xy)\,(3Xy^2)-(2Xy)\,(5y)$
$$=8X^3y+6X^2y^3-10Xy^2$$

■ Self-Evaluation ③

Simplify to the simplest form:

$3\,(5X^2+3X-2)-15X^2$

Example ③

Simplify the expression to the simplest form:

$4\,(3X^2+5X)-X\,(X^2-7X+8)$

✓

$4\,(3X^2+5X)-X\,(X^2-7X+8)$

$=(4)\,(3X^2)+(4)\,(5X)+(-X)\,(X^2)-(-X)\,(7X)+(-X)\,(8)$
$\qquad\qquad\qquad\qquad\qquad\qquad$ (Distributive property)

$=12X^2+20X+(-X^3)-(-7X^2)+(-8X)$ $\quad$ (Multiplication of terms)

$=12X^2+20X-X^3+7X^2-8X$

$=-X^3+19X^2+12X$ $\qquad\qquad\qquad\qquad$ (Addition of like terms)

Find the area of the rectangle in terms of x,
then calculate the numerical value of the area when $x = 2$.

$$5x$$

$$x^2 + 3x + 4$$

Example 4

The following figure represents a swimming pool surrounded by a walkway, with the dimensions indicated in the figure. Find the area of the walkway in terms of x

✅
- Area of the swimming pool and the walkway together
  $= 4 x (5 x + 8) = 20 x^2 + 32 x$
- Area of the swimming pool $= (3 x)(6 x) = 18 x^2$
- Area of the walkway
  $=$ Area of the swimming pool and the walkway $-$ Area of the swimming pool
  $= 20 x^2 + 32 x - 18 x^2 = 2 x^2 + 32 x$
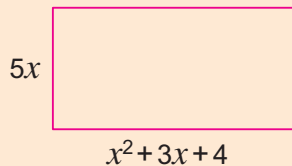
A rectangular garden has a length of 50 meters and a width of x meters. If its length is increased by x meters, its area increases by 100 square meters, what is the value of x?

Example 5

A water tank in the shape of a rectangular prism has dimensions as shown in the figure and is filled with water. If the water leaks out until the height of the water in it becomes $(x + 3)$ meters, what is the value of x if the volume of water leaked from the tank is 27 cubic meters?

$(2x + 3) \, m$

$x \, m$

$x \, m$
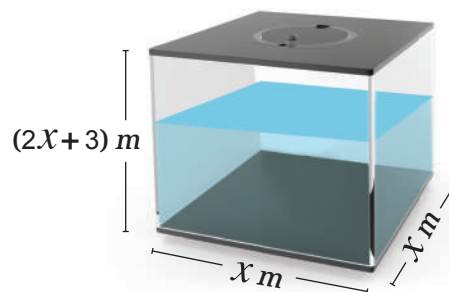
✅
∵ Volume of water leaked $=$ Volume of the tank $-$ Volume of the remaining water in the tank

∴ $x \times x \times (2x + 3) - x \times x \times (x + 3) = 27$

∴ $x^2 (2x + 3) - x^2 (x + 3) = 27$ ∴ $2x^3 + 3x^2 - x^3 - 3x^2 = 27$

∴ $x^3 = 27$ ∴ $x = \sqrt[3]{27} = 3$

Another Solution:

The height of the empty part of the tank due to the leak

$=$ Height of the tank $-$ Height of the remaining water

∴ $(2x + 3) - (x + 3) = 2x + 3 - x - 3 = x$

∵ Volume of water leaked $= 27$

∴ $x \times x \times x = 27$ ∴ $x^3 = 27$ ∴ $x = \sqrt[3]{27} = 3$

▶ **Choose the correct answer from the given ones:**

① $(2\,X)\,(3\,X) =$ ········

(a) $5\,X$

(b) $6\,X$

(c) $5\,X^2$

(d) $6\,X^2$

② $(-3\,X^2)\,(4\,X^3) =$ ········

(a) $-12\,X^5$

(b) $12\,X$

(c) $-12\,X^6$

(d) $12\,X^5$

③ $2\,(X+3) =$ ········

(a) $2\,X^2 + 6\,X$

(b) $2\,X + 3$

(c) $2\,X + 6$

(d) $X + 6$

④ $X\,(X-1) + X =$ ········

(a) $X\,(2\,X-1)$

(b) $2\,X^2$

(c) X^2

(d) $X^2 - X$

▶ **Spot the mistake:**

⑤ If the length of a rectangle exceeds its width by 3 units and the width of the rectangle equals X units, what is the area of the rectangle? Which of Hamza and Alia followed the correct method in the solution? Discuss.

Hamza's Solution	Alia's Solution
$\because$ Width of the rectangle $= X$ length units	$\because$ Width of the rectangle $= X$ length units
$\therefore$ Length of the rectangle $= (X+3)$ length units	$\therefore$ Length of the rectangle $= (X+3)$ length units
$\therefore$ Area of the rectangle $= X\,(X+3)$	$\therefore$ Area of the rectangle $= X\,(X+3)$
$\qquad = X^2 + 3$ area units	$\qquad = X^2 + 3\,X$ area units

▶ **Find the result of each of the following:**

⑥ $(-5\,X^2\,y^2)\,(3\,X\,y)$

⑦ $(a^2\,b^3)\,(4\,a^4\,b^3)$

⑧ $(-4\,m)\,(-6\,m^5)$

⑨ $(7\,P^3\,S^2\,r)\,(-4\,P^2\,r)$

▶ **Find the result of each of the following:**

⑩ $-4\,a\,(3\,a-2)$

⑪ $8\,X^2\,(2\,X^3 - 3\,X^2 - X + 4)$

⑫ $2\,X\,(4\,X^2 - X\,y + 5)$

⑬ $-h\,g\,(3\,h^2\,g - 4\,h\,g^2 + 2\,h\,g)$

▶ **Simplify each of the following to the simplest form:**

⑭ $2\,X\,(X^2 - 2\,X - 3) - X^2\,(3\,X - 5)$

⑮ $X\,(X^2 - X - 1) + 3\,(2\,X^2 + X + 1)$

⑯ **Simplify to the simplest form :** $2\,X\,(3\,X - 1) + 3\,X\,(X + 2)$

then find the numerical value of the resulting expression when $X = 1$.

⑰ **Solve the following equation in** $\mathbb{Z}$ **:** $X\,(X-2) + 2\,(X-2) = 0$

⑱ **Constructions:** A carpenter wants to create a wooden roof in the shape of a rectangle with dimensions $(6X^2 + X + 3)$, $(5X)$ length units. Find the area of the roof in terms of X, then find the numerical value of the area when $X = 1$.

Each craft has its risks that require training to avoid them

▶ **Geometry:** Find in the simplest form the algebraic expression that represents the area of the shaded part in each of the following :

⑲

$2x$ — y
x

⑳

$2x$ — 2 — x
$X+6$

㉑

$3X+4$
$5X$ $2X$
$2X^2+3X+5$

㉒

$3X$ $2X$ $2X+3$
X^2+3X+9
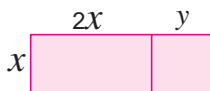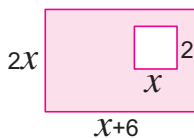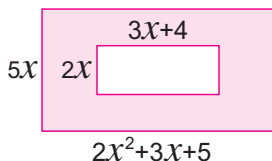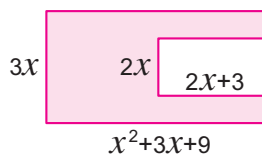
㉓ **Geometry:** Find the area of triangle ABC in terms of X, then calculate the numerical value of its area when $X = 3$.

A
$2X$
B — X^2+2X+3 — C

$3x$
$2x$
$3x+2$
$10x$
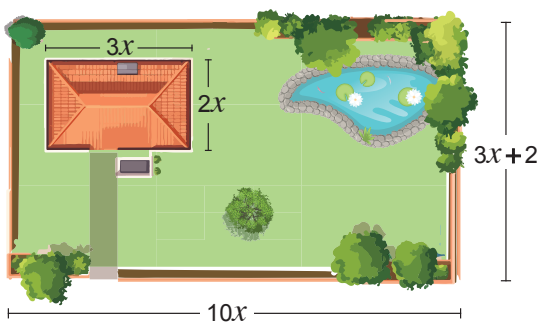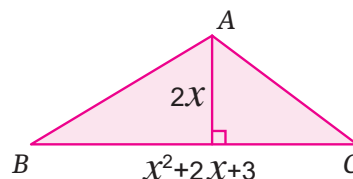
㉔ **Architectural Engineering:** The opposite figure represents a plan for a house surrounded by a garden. The dimensions of the house and the garden are illustrated in the figure. Find the area of the garden in terms of X.

Creative Thinking

㉕ Two positive numbers a, b, and if a is increased by b, then their product increases by 9. Find the number b.
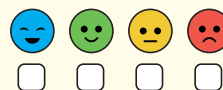
Multiplying Algebraic Expressions

■ Learning Outcomes

- Multiply a binomial by another binomial.
- Learn the special cases of multiplying binomials.
- Multiply a binomial by an algebraic expression consisting of more than two terms.

■ Vocabulary

- Algebraic Expression
- Multiplication
- Product
- Distributive Property

Get Ready!

Omar has a rectangular garden with a length of $2x$ meters and a width of x meters.

He decided to add 7 meters to the length and 3 meters to the width in order to plant more trees.

What is the algebraic expression that represents the increase in the area of the garden?

In this lesson, you will learn how to multiply algebraic expressions, which will help you solve such real-life problems.

Think & Discuss!

A square with a side length of x cm; if the length of one of its sides is increased by 1 cm, and the length of an adjacent side is decreased by 1 cm, this square becomes a rectangle. Which is larger: the area of the square or the area of the rectangle?

Bassem's Opinion	**Mariam's Opinion**
The area of the square is larger.	*The area of the rectangle is larger.*

Which of the two opinions do you think is correct? Discuss.

Learn!

Multiplying a Binomial by Another Binomial

When multiplying a binomial by another binomial, multiply each term of the first binomial by the two terms of the second binomial using the distributive property.

- $(x + a)(y + b) = x(y + b) + a(y + b)$

$$= xy + bx + ay + ab$$

Product of the first two terms	Product of the extremes	Product of the means	Product of the last two terms

■ Note that

When multiplying a binomial by another binomial, the result is an algebraic expression consisting of 4 terms, which may include like terms that should be added to be in simplest form.

■ Modelling

The product of :
$(x + 3)(x + 2)$

can be found geometrically using the rectangle area model as follows:
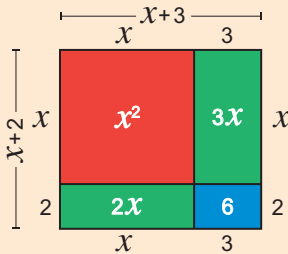
$(x + 3)(x + 2)$
$= x^2 + 5x + 6$

■ Self-Evaluation ①

Find the product of each of the following:

❶ $(x + 1)(x - 6)$

❷ $(3x - 1)(2x + 5)$

❸ $(m + 4n)(2m - n)$

• $(x + a)(x + b) = x(x + b) + a(x + b)$

$$= x^2 + bx + ax + ab$$

$$= x^2 + (b + a)x + ab$$

| Product of the first two terms | Sum of the products of the extremes and means | Product of the last two terms |

■ Multiplying by inspection

Multiplication can be done directly as follows:

$$= x^2 + 5x + 6$$

For example:

$(x + 3)(x + 2) = x(x + 2) + 3(x + 2)$
$$= x^2 + 2x + 3x + 6$$
$$= x^2 + 5x + 6$$
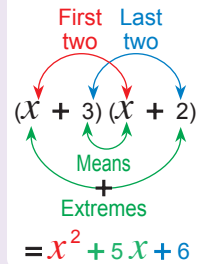
The vertical method

The vertical method can be used to find the product as shown in the opposite figure.
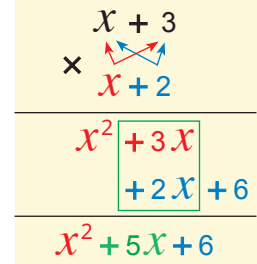
Example ① 1

Find the product of each of the following:

❶ $(x - 4)(x + 5)$ ❷ $(a - 3b)(2a - 5b)$

✓

① $(x - 4)(x + 5) = x^2 + 5x - 4x - 20$
$$= x^2 + x - 20$$

► Try to do the multiplication by inspection.

■ The vertical method
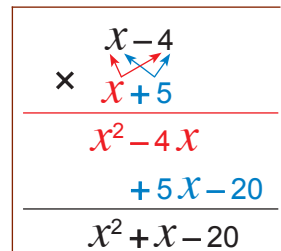
② $(a - 3b)(2a - 5b) = 2a^2 - 5ab - 6ab + 15b^2$
$$= 2a^2 - 11ab + 15b^2$$

► Try to do the multiplication using the vertical method, and by inspection.
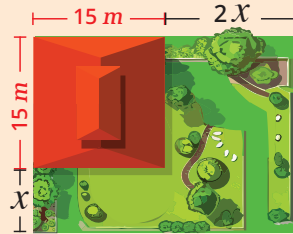
■ **Modelling**

$(a + b)^2$ geometrically represents the area of a square with side length $(a + b)$.
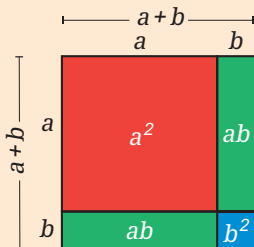
$(a + b)^2 = a^2 + 2\, ab + b^2$

■ **Discuss with your classmates**

• Is $2 - X = X - 2$?
• Is $(2 - X)^2 = (X - 2)^2$?

■ **Self-Evaluation** ③

A square with side length $(2X + 3)$ cm. Find its area in terms of X

Example 2

Asmaa covered a part of the floor of her room with a rectangular coloured carpet of dimensions 4 meters and 6 meters, leaving spaces from the floor such that the dimensions between the four walls and the edges of the carpet are equal, as shown in the opposite figure. Determine the uncovered area of the room's floor in terms of X.

✓

∵ The uncovered area of the room's floor = The area of the room's floor
$-$ The area of the carpet

∴ $(2X + 6)(2X + 4) - 6 \times 4 = 4X^2 + 8X + 12X + 24 - 24$
$$= 4X^2 + 20X$$

Thus, the uncovered area of the room's floor is $(4X^2 + 20X)$ square meters.

Special Cases

1 **Expansion of the square of a binomial**

$(a + b)^2 = (a + b)(a + b)$

First term, Second term

$= a(a + b) + b(a + b)$

$= a^2 + ab + ba + b^2$

$= a^2 + 2\,ab + b^2$

Square of the first term | Twice the product of the 1st term by the 2nd term | Square of the second term

For example: $(X + 3)^2 = X^2 + 2 \times X \times 3 + 3^2 = X^2 + 6X + 9$

$(a - b)^2 = (a - b)(a - b)$

$= a(a - b) - b(a - b)$

$= a^2 - ab - ba + b^2$

$= a^2 - 2\,ab + b^2$

For example: $(X - 2)^2 = X^2 - 2 \times X \times 2 + 2^2 = X^2 - 4X + 4$

Example 3

Find the expansion of each of the following:

① $(4 - X)^2$ ② $(2X + 5)^2$

✓

① $(4 - X)^2 = 4^2 - 2 \times 4 \times X + X^2 = 16 - 8X + X^2$

② $(2X + 5)^2 = (2X)^2 + 2 \times 2X \times 5 + 5^2 = 4X^2 + 20X + 25$

■ **Note that**

When multiplying the sum of two terms by the difference between them, note that the last terms in the two expressions are additive inverses of one another; therefore, the product of these last terms is always negative.

2 | **The product of the sum of two terms and the difference between them**

$$(a + b)\,(a - b) = a\,(a - b) + b\,(a - b)$$
$$= a^2 - ab + ba - b^2 = a^2 \quad - \quad b^2$$

| The square of the first term | The square of the second term |

For example: $(X + 3)\,(X - 3) = X^2 - 3^2 = X^2 - 9$

■ **Self-Evaluation** ④

Find in the simplest form:

① $(a - 4)\,(a + 4)$

② $(X + 5)\,(5 - X)$

③ $(3\,b + 2)\,(3\,b - 2)$

④ $(a + b)^2 - (a + b)\,(a - b)$

Example 4

Find in the simplest form each of the following:

① $(3\,X + 2)\,(3\,X - 2)$ ② $(4\,a - 3\,b)^2 - (4\,a - 3\,b)\,(4\,a + 3\,b)$

✅

① $(3\,X + 2)\,(3\,X - 2) = (3\,X)^2 - 2^2 = 9\,X^2 - 4$

② $(4\,a - 3\,b)^2 - (4\,a - 3\,b)\,(4\,a + 3\,b)$

$= (4\,a)^2 - 2\,(4\,a)\,(3\,b) + (3\,b)^2 - \left((4\,a)^2 - (3\,b)^2\right)$

$= 16\,a^2 - 24\,ab + 9\,b^2 - 16\,a^2 + 9\,b^2 = -24\,ab + 18\,b^2$

Multiplying a binomial by an algebraic expression consisting of more than two terms

■ **Self-Evaluation** ⑤

Find in the simplest form the product of each of the following:

① $(X + 2)\,(X^2 - X + 3)$

② $(2\,X - 3)\,(6\,X + 4\,X^2 + 9)$

■ **Note that**

When using the vertical method for multiplying algebraic expressions, it is better to arrange the expression in ascending or descending order according to the exponent of the variable in the expression.

Example 5

Find in the simplest form the product of each of the following:

① $(X - 2)\,(X^2 - 3\,X + 5)$ ② $(2\,X + 3)\,(1 + 3\,X^2 - 2\,X)$

✅

Use the distributive property:

■ **The vertical method**

① $(X - 2)\,(X^2 - 3\,X + 5)$

$= X\,(X^2 - 3\,X + 5) - 2\,(X^2 - 3\,X + 5)$

$= X^3 - 3\,X^2 + 5\,X - 2\,X^2 + 6\,X - 10$

$= X^3 - 5\,X^2 + 11\,X - 10$

$$
\begin{array}{r}
X^2 - 3\,X + 5 \\
X - 2 \\
\hline
X^3 - 3\,X^2 + 5\,X \\
-2\,X^2 + 6\,X - 10 \\
\hline
X^3 - 5\,X^2 + 11\,X - 10
\end{array}
$$

② $(2\,X + 3)\,(1 + 3\,X^2 - 2\,X) = 2\,X\,(1 + 3\,X^2 - 2\,X) + 3\,(1 + 3\,X^2 - 2\,X)$

$= 2\,X + 6\,X^3 - 4\,X^2 + 3 + 9\,X^2 - 6\,X$

$= 6\,X^3 + 5\,X^2 - 4\,X + 3$

► Try to do the multiplication using the vertical method.

First | **Measuring Conceptual Understanding** ⏱

▶ **Choose the correct answer from the given ones:**

① What is the number of terms in the expression resulting from the product of $(X - 3)(X + 4)$ in the simplest form?

    (a) 1   (b) 2   (c) 3   (d) 4

② If $(X - 5)(X + 2) = X^2 + bX + c$, what is the value of c?

    (a) -10 (b) -7 (c) 7   (d) 10

③ If $(3X - 7)^2 = aX^2 + bX + c$, what is the value of b?

    (a) -42 (b) -21 (c) 21   (d) 42

④ If $(X - 5)(X + 5) = X^2 + bX + c$, what is the value of b?

    (a) -25 (b) -10 (c) 0   (d) 10

▶ **Spot the mistake:**

⑤ Both Najwa and Ahmed expand the binomail of $(2X - 3)^2$, which of the two solutions is correct? Discuss.

Najwa's Solution:

$(2x - 3)^2 = (2x)^2 - 2(2x)(3) + 3^2$
$= 4x^2 - 12x + 9$

Ahmed's Solution:

$(2x - 3)^2 = (2x)^2 - 3^2$
$= 4x^2 - 9$

Second | **Applying Scientific Concepts** 📝

▶ **Choose the correct answer from the given ones:**

⑥ If $X + y = 15$, $X - y = 5$, what is the value of $x^2 - y^2$?

    (a) 75   (b) 20   (c) 10   (d) 3

⑦ If $(X + y)^2 = 16$, $Xy = 3$, what is the value of $x^2 + y^2$?

    (a) $5\frac{1}{3}$ (b) 10   (c) 13   (d) 48

⑧ What is the result of subtracting $(a + b)^2$ from $(a - b)^2$?

    (a) $2ab$ (b) $-2ab$ (c) $-4ab$ (d) $4ab$

▶ **Find the product of each of the following:**

⑨ $(a + 3)(a + 4)$

⑩ $(X + 3)(5 - X)$

⑪ $(X - 7)(2X - 1)$

⑫ $(3X + 1)(X - 3)$

⑬ $(3 - 2m)(m - 4)$

▶ **Find the expansion of each of the following:**

⑭ $(X + 7)^2$

⑮ $(X - 6)^2$

⑯ $(2X - 9)^2$

▶ **Find the product of each of the following:**

⑰ $(4X - 3)(4X + 3)$

⑱ $\left(\frac{1}{2}X + 1\right)\left(\frac{1}{2}X - 1\right)$

⑲ $(7 - 5X)(5X + 7)$

▶ **Find the product of each of the following:**

(20) $(2X - 1)(X^2 - 3X + 4)$

(21) $(X + 3)(X + 2)^2$

(22) $(X - 2)(X + 2)(X^2 + 4)$

(23) **Simplify the expression to the simplest form:** $(2n - 1)^2 - (2n + 1)(2n - 1)$
Then find the numerical value of the resulting expression when $n = -3$

▶ **Find the solution set for each of the following equations in Q:**

(24) $(X - 2)(X + 2) = 5$

(25) $(X - 3)(X^2 + 3X + 9) = 0$

Third | **Analysis and Subjects Integration**

(26) **Geometry:** A square with a side length of $(X + 3)$ length units. Find its surface area in terms of X, then find the numerical value of the area when $X = 7$

(27) **Geometry:** A rectangle with a length of $(2X + 5)$ length units, and its length exceeds its width by 4 length units. Find its surface area in terms of X, then find the numerical value of the area when $X = 3$

(28) **Sports:** The opposite figure shows a swimming pool with dimensions of 10 meters and 8 meters, surrounded by a walkway of width X meters. Write an algebraic expression representing the area of the pool and the walkway combined in terms of X.

10 m

8 m

X

X

Swimming enhances physical fitness.

X

X

6 m

15 m

Flowers add an aesthetic touch to gardens and homes, and help improve mood and mental health.
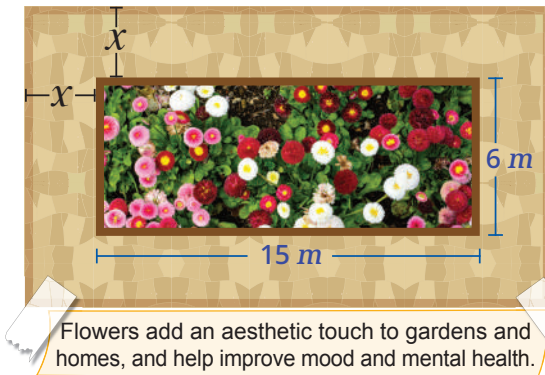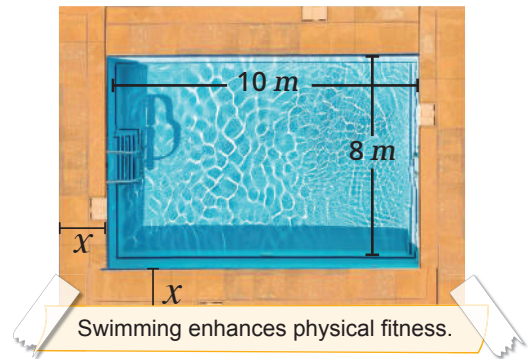
(29) **Agriculture:** The opposite figure shows a pathway of width X meters around a rectangular flowerbed of length 15 meters and width 6 meters. What is the area of this pathway in terms of X ?
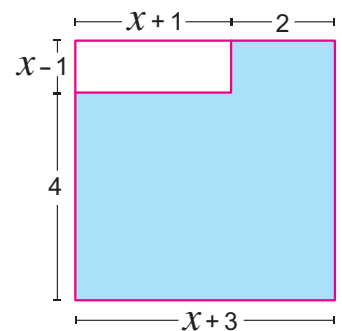
(30) **Geometry:**
Find the area of the shaded part in the opposite figure in terms of X, then find the numerical value of the area when $X = 2$

Creative Thinking

(31) If the length of each side of a square increases by 2 cm, the area increases by 24 square centimeters. What was the length of a side of the square before the increase?

$X + 1$ 2

$X - 1$

4

$X + 3$

Lesson 2 - 4

Dividing an Algebraic Term or an Algebraic Expression by an Algebraic Term

Get Ready!

The Avenue of Sphinxes is located in Luxor city; it is originally a royal avenue in the shape of a rectangle connecting the "Luxor Temple" and the "Karnak Temple".
It is named so because it is adorned on both sides with statues in the form of sphinxes, and it was used by the kings of ancient Egypt during celebrations and religious rituals. Assuming that the area of this avenue equals ($7 x y^2 + 11 x^3 y$) square meters, and the width of the avenue is ($x y$) meters, what is the length of this avenue in terms of x and y?
Then find the length and width when $x = 4$, $y = 19$.

Avenue of Sphinxes in Luxor

In this lesson, you will learn how to divide algebraic terms and divide an algebraic expression by an algebraic term, which can help you solve such problems.

Think & Discuss!

You know from multiplying algebraic terms that: $(3 x y^2) (2 x^2 y^3) = 6 x^3 y^5$

Can you calculate: $\dfrac{6 x^3 y^5}{3 x y^2}$? And what is the numerical value of the quotient when $x = 3$, $y = 2$?

Learn!

Dividing an Algebraic Term by Another Algebraic Term

When dividing an algebraic term by another algebraic term, divide the coefficients and subtract the exponents of the variables that have the same base.

Subtract the exponents

For example: $(- 14 x^6) \div (2 x^2) = - 7 x^4$

Divide the coefficients

Note that division by zero is undefined, and therefore all problems involving variables you have to ensure that the divisor does not equal zero.

■ **Hint**

In both the dividend and the divisor containing the same variable with the same exponent, it should be cancelled out, since their quotient equals 1, and then continue with the division. For example, in the problem ❸

$$\dfrac{15\,a^3\,\cancel{b^2}\,\cancel{c}}{3\,a^2\,\cancel{b^2}\,\cancel{c}} = \dfrac{15\,a^3}{3\,a^2} = 5\,a$$

Example 1

Find the quotient of each of the following:

① $\dfrac{9\,x^4}{-3\,x^3}$ **②** $\dfrac{-12\,x^5\,y^2}{-4\,x^2\,y}$ **③** $\dfrac{15\,a^3\,b^2\,c}{3\,a^2\,b^2\,c}$

✅

① $\dfrac{9\,x^4}{-3\,x^3} = -3\,x$ ② $\dfrac{-12\,x^5\,y^2}{-4\,x^2\,y} = 3\,x^3\,y$

③ $\dfrac{15\,a^3\,b^2\,c}{3\,a^2\,b^2\,c} = 5\,a$

Dividing an algebraic expression by an algebraic term

From your study of proper fractions, you know that:

$$\dfrac{a+b}{c} = \dfrac{a}{c} + \dfrac{b}{c} \qquad \dfrac{a-b}{c} = \dfrac{a}{c} - \dfrac{b}{c}$$

And you will use the same method when dividing an algebraic expression by an algebraic term, such that you can divide each term of the expression by this term.

For example:

- $\dfrac{10\,x^2 + 8\,x}{2\,x} = \dfrac{10\,x^2}{2\,x} + \dfrac{8\,x}{2\,x} = 5\,x + 4$

- $\dfrac{9\,x^3 - 6\,x^2}{3\,x} = \dfrac{9\,x^3}{3\,x} - \dfrac{6\,x^2}{3\,x} = 3\,x^2 - 2\,x$

Example 2

Find the quotient of each of the following:

① $\dfrac{18\,x^3 + 12\,x^2 - 6\,x}{-6\,x}$ **②** $\dfrac{3\,a\,b^2 + 9\,a^2\,b - 6\,a^2\,b^2}{3\,a\,b}$

③ $\dfrac{-8\,x^2\,(4\,x^2 - 2\,x - 6)}{4\,x}$

✅

① $\dfrac{18\,x^3 + 12\,x^2 - 6\,x}{-6\,x} = \dfrac{18\,x^3}{-6\,x} + \dfrac{12\,x^2}{-6\,x} + \dfrac{-6\,x}{-6\,x}$

$$= -3\,x^2 - 2\,x + 1$$

② $\dfrac{3\,a\,b^2 + 9\,a^2\,b - 6\,a^2\,b^2}{3\,a\,b} = \dfrac{3\,a\,b^2}{3\,a\,b} + \dfrac{9\,a^2\,b}{3\,a\,b} - \dfrac{6\,a^2\,b^2}{3\,a\,b}$

$$= b + 3\,a - 2\,a\,b$$

■ Diversified Strategies

You can solve problem **③** by dividing $(-8X^2)$ by $(4X)$ first before expanding the brackets, then performing the multiplication as follows:

$$\frac{-8X^2(4X^2-2X-6)}{4X}$$

$$=-2X(4X^2-2X-6)$$

$$=-8X^3+4X^2+12X$$

③ $\dfrac{-8X^2(4X^2-2X-6)}{4X} = \dfrac{-32X^4+16X^3+48X^2}{4X}$

$$= \frac{-32X^4}{4X} + \frac{16X^3}{4X} + \frac{48X^2}{4X}$$

$$= -8X^3 + 4X^2 + 12X$$

Example ③

If the area of the opposite triangle is $(15X^4 + 6X^3 + 9X^2)$ square units, find the length of its base in terms of X if the height corresponding to this base is $(3X^2)$ length units, then calculate the numerical value of the base length when $X = 3$.

$3x^2$

✅
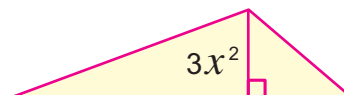
∵ Area of the triangle $= \dfrac{1}{2}$ the length of the base × the corresponding height

∴ Length of the base $= \dfrac{\text{Twice the area of the triangle}}{\text{height}}$

Thus, the length of the base in terms of X equals:

$$\frac{2(15X^4 + 6X^3 + 9X^2)}{3X^2} = \frac{30X^4 + 12X^3 + 18X^2}{3X^2}$$

$$= 10X^2 + 4X + 6$$

The numerical value of the base length in length units equals:

$$10 \times 3^2 + 4 \times 3 + 6 = 90 + 12 + 6 = 108$$

■ Self-Evaluation ③

If the area of a rectangle equals $(4X^4 + 8X^3 + 12X^2)$ square units, and one of its dimensions is $4X^2$ length units, find the other dimension in terms of X.

■ Self-Evaluation ④

A bakery prepares $(10X^3 + 15X^2 + 5X)$ pieces of biscuit and places them in boxes such that each box contains $(5X)$ pieces. Determine the number of boxes required by the bakery to package the biscuits in terms of X.

Example ④

A garden in the shape of a rectangle has dimensions $(4X)$ and $(12X^3 + 18X^2 + 30X)$ length units. Walid wants to divide it into square plots, each with an area of $(4X^2)$ square units. Find the number of plots in terms of X.

✅

The area of the garden in square units equals:

$$4X(12X^3 + 18X^2 + 30X) = 48X^4 + 72X^3 + 120X^2$$

The number of plots in terms of X equals:

$$\frac{48X^4 + 72X^3 + 120X^2}{4X^2} = 12X^2 + 18X + 30$$

Lesson Assessment

▶ **Choose the correct answer from the given ones:**

(1) $24\,x^3 \div (-6\,x^2) = $ ……………

   (a) -4 (b) $-4\,x$

   (c) $-4\,x^5$ (d) $-4\,x^2$

(2) If $\dfrac{8\,x^2}{a} = 1$, what is the value of a?

   (a) -1 (b) 1

   (c) $-8\,x^2$ (d) $8\,x^2$

(3) $\dfrac{a+b}{c} = $ ……………

   (a) $a + \dfrac{b}{c}$ (b) $\dfrac{a}{c} + b$

   (c) $\dfrac{a}{c} + \dfrac{b}{c}$ (d) $\dfrac{ab}{c}$

(4) $(x^3 + x^2) \div x^2 = $ ……………

   (a) 0 (b) x

   (c) $x + 1$ (d) $2\,x + 1$

(5) $\dfrac{3\,x^2 - 6\,x}{3\,x} = $ ……………

   (a) $-x$ (b) $-x^2$

   (c) $x^2 - 2\,x$ (d) $x - 2$

(6) ……………… $\div (-2\,x^2\,y) = 12\,x\,y^2$

   (a) $6\,x\,y$ (b) $-6\,x\,y$

   (c) $24\,x^3\,y^3$ (d) $-24\,x^3\,y^3$

▶ **Find the quotient of each of the following:**

(7) $(27\,x^3) \div (9\,x)$

(8) $(-18\,x^2) \div (-3\,x^2)$

(9) $(x - x^2 - x^3) \div (-x)$

(10) $(2\,x - 4\,x^2 + 8\,x^3) \div (2\,x)$

▶ **Simplify each of the following to the simplest form:**

(11) $\dfrac{x^2}{-x} + \dfrac{-4\,x}{x} - \dfrac{3\,x^3}{x^2}$

(12) $\dfrac{28\,x^2 - 42\,x}{7\,x} + \dfrac{14\,x^2 - 35\,x}{-7\,x}$

(13) $\dfrac{2\,x\,(6\,x^2 - 2\,x + 8)}{4\,x}$

(14) $\dfrac{48\,x^4 - 144\,x^3 - 96\,x^2}{-6\,x \times 8\,x}$

(15) If $(30\,x^2\,y^3 - 15\,x\,y) \div (-3\,x) = n\,x\,y^3 + 5\,y$,

   what is the value of n?

▶ Spot the mistake:

(16) Both Samar and Anas find the quotient of: $\dfrac{4X^2 - 6X}{-2X}$,

which of them followed the correct method to solve the question? Discuss.

Samar's Solution:

$$\frac{4X^2 - 6X}{-2X} = \frac{4X^2}{-2X} + \frac{-6X}{-2X} = -2X + 3$$

Anas's Solution:

$$\frac{4X^2 - 6X}{-2X} = \frac{4X^2}{-2X} - \frac{-6X}{-2X} = -2X - 3$$

Third | Analysis and Subjects Integration

(17) **Fuel Consumption:** A car's fuel consumption during a journey equals ($9X^3 + 18X^2 + 27X$) liters, and the car consumes $9X$ liters of fuel per kilometer, what is the number of kilometers the car covers during the journey in terms of X?

Reducing fuel consumption keeps the environment clean.

(18) **Education:** If the number of students in a certain school equals ($25X^4 + 50X^3 + 75X^2$) and the number of students in each class equals ($25X$), what is the number of classes in the school in terms of X?

(19) **Wall Painting:** When painting a wall with an area of ($12X^3 + 72X^2 + 48X$) square meters, if each can of paint covers an area of ($12X$) square meters, how many cans of paint are needed in terms of X?

Green spaces play a significant role in purifying the air and improving an individual's mental state.

(20) A rectangular garden with an area of ($15a^3b^2 + 20a^2b^3 - 25a^2b^2$) square meters, and a length of ($5a^2b^2$) meters. Find its width in terms of a and b, and if $a = 2$, $b = 1$, find the numerical value of the width of the garden.

Creative Thinking

(21) The volume of a cuboid is ($24X^3 + 32X^2y$) cubic centimeters, and its base is in the shape of a square with a side length of ($2X$) cm. Find its height in terms of X and y, then find the numerical value of the height when $X = 2$, $y = 1$.

Evaluate your understanding!

How well do you understand dividing an algebraic term or an algebraic expression by an algebraic term? Tick the right box.

Dividing Algebraic Expressions

■ **Learning Outcomes**

• Divide an algebraic expression by another algebraic expression consisting of two terms.

• Use the division of algebraic expressions in problem-solving.

■ **Vocabulary**

• Division
• Algebraic Expression
• Dividend
• Divisor
• Quotient

■ **Remember**

Long division in numbers :

$972 \div 4$

Steps

► Divide

► Multiply

► Subtract

► Bring down

$$
\begin{array}{r}
243 \\
4\overline{)972} \\
-8 \\
\hline
17 \\
-16 \\
\hline
12 \\
-12 \\
\hline
0
\end{array}
$$

Get Ready!

The Bibliotheca Alexandrina is one of the most important libraries and cultural centers in the world. Suppose the number of books in one of the library's halls is ($8x^3 + 1$) books, and the number of books on each shelf is ($2x + 1$) books, what is the number of shelves within that hall?

In this lesson, you will learn how to divide an algebraic expression by another, which will help you solve such problems.

Bibliotheca Alexandrina

Think & Discuss!

The figure opposite represents a model of a piece of land in the shape of a rectangle with an area of ($x^2 + 7x + 12$) square meters and a width of ($x + 3$) meters.

Can you find the length of the piece of land just by inspection? And can you use other methods to find the length of the piece of land?

x $4x$ x^2

3 12 $3x$

Learn!

From the multiplication of algebraic expressions, you know that :

$$(x - 5)(x + 4) = x^2 - x - 20$$

Since division is the inverse operation of multiplication, you can deduce the following :

► $(x^2 - x - 20) \div (x - 5) = x + 4$

► $(x^2 - x - 20) \div (x + 4) = x - 5$

And division can be written as follows :

Divisor → $x - 5\overline{)x^2 - x - 20}$ → Dividend, with Quotient $x + 4$

Divisor → $x + 4\overline{)x^2 - x - 20}$ → Dividend, with Quotient $x - 5$

Note that in all problems involving variables, the divisor does not equal zero.

Example 1

Find the quotient of: $(x^2 - x - 20)$ divided by $(x - 5)$

✓

▶ Divide x^2 by x, the quotient is x

▶ Multiply x by $(x - 5)$, the product is

▶ Subtract $(x^2 - 5x)$ from $(x^2 - x - 20)$, the difference is

▶ Repeat the previous steps until the remainder equals zero, hence the quotient is $(x + 4)$

$$
\begin{array}{r}
x + 4 \\
x - 5 \overline{\smash{)}\,x^2 - x - 20} \\
\underline{x^2 - 5x} \\
4x - 20 \\
\underline{4x - 20} \\
0 \quad 0
\end{array}
$$

Note that: $(x^2 - x - 20)$ is the dividend, $(x - 5)$ is the divisor, and $(x + 4)$ is the quotient.

Example 2

Find the quotient of: $(2x + x^3 - 12)$ divided by $(x - 2)$

✓ Write the dividend after arranging its terms in a descending order according to the powers of x as follows:

$$x^3 + 2x - 12$$

Note that there is no term including x^2, therefore, leave a blank space for it.

▶ The quotient is $(x^2 + 2x + 6)$

$$
\begin{array}{r}
x^2 + 2x + 6 \\
x - 2 \overline{\smash{)}\,x^3 \qquad\, + 2x - 12} \\
\underline{x^3 - 2x^2} \\
2x^2 + 2x - 12 \\
\underline{2x^2 - 4x} \\
6x - 12 \\
\underline{6x - 12} \\
0 \quad 0
\end{array}
$$

Example 3

If $(x - 3)$ is one factor of the expression $(3x^2 - 14x + 15)$, **find the other factor.**

✓

The other factor is the quotient of $(3x^2 - 14x + 15)$ divided by $(x - 3)$

Thus, the other factor is $(3x - 5)$

$$
\begin{array}{r}
3x - 5 \\
x - 3 \overline{\smash{)}\,3x^2 - 14x + 15} \\
\underline{3x^2 - 9x} \\
-5x + 15 \\
\underline{-5x + 15} \\
0 \quad 0
\end{array}
$$

If the expression
$(x^3 - x^2 - 4x - m)$
is divisible by $(x - 3)$,
find the value of m.

Example 4

If the expression $(x^3 + x^2 + a)$ is divisible by $(x - 2)$, find the value of a.

✓

∵ The dividend is divisible
   by the divisor

∴ $a + 12 = 0$

∴ $a = -12$

$$
\begin{array}{r}
x^2 + 3x + 6 \\
x - 2 \overline{\smash{\big)}\, x^3 + x^2 \qquad\quad + a} \\
x^3 - 2x^2 \\
\hline
3x^2 \qquad\quad + a \\
3x^2 - 6x \\
\hline
6x + a \\
6x - 12 \\
\hline
a + 12
\end{array}
$$

A theatre sells tickets with
total revenues of
$(4x^2 + 16x + 12)$ pounds.
If the price of each ticket is
$(4x + 4)$ pounds,
determine the number of
tickets sold in terms of x.

Example 5

Link to Life: A rectangular apartment
has a floor area of
$(x^3 + 15x^2 + 51x + 10)$ square
meters. If the width of the apartment
floor is $(x + 10)$ meters,

find the length of the apartment floor in
terms of x,

then find the numerical value of the
perimeter of the apartment floor,

when $x = 2$.

$(x + 10)$

✓

∵ Length of the apartment floor
   $= \dfrac{\text{area}}{\text{width}}$

∴ Length of the apartment floor
   $= (x^2 + 5x + 1)$ meters.

When $x = 2$:

∴ Width $= 2 + 10 = 12$ meters.

   Length $= 4 + 10 + 1 = 15$ meters.

∴ Perimeter of the apartment floor
   $= 2 (12 + 15)$
   $= 54$ meters.

$$
\begin{array}{r}
x^2 + 5x + 1 \\
x + 10 \overline{\smash{\big)}\, x^3 + 15x^2 + 51x + 10} \\
x^3 + 10x^2 \\
\hline
5x^2 + 51x + 10 \\
5x^2 + 50x \\
\hline
x + 10 \\
x + 10 \\
\hline
0 \qquad 0
\end{array}
$$

▶ **Choose the correct answer from the given ones:**

① If $\dfrac{2X + a}{X + 3} = 2$, what is the value of a?

  (a) 2   (b) 3   (c) 5   (d) 6

② If $\dfrac{3X + 15}{X - a} = 3$, what is the value of a?

  (a) -5 (b) -3 (c) 3   (d) 5

③ If $\dfrac{X - 2}{2 - X} = a$, what is the value of a?

  (a) -2 (b) -1 (c) 1   (d) 2

④ If the quotient of $(X^2 - 2X - 35)$ divided by $(X + 5)$ is $(X + b)$, what is the value of b?

  (a) -7 (b) -5 (c) 5   (d) 7

⑤ If the quotient of $(X^3 - 4X)$ divided by $(X - 2)$ is $(aX + X^2)$, what is the value of a?

  (a) -4 (b) -2 (c) 2   (d) 4

▶ **Spot the mistake:**

⑥ Both Omar and Khaled divided $(X^2 + 3X - 4)$ by $(X + 4)$, and the quotient in each of their solutions was $(X - 1)$. Is each of their solutions correct? Discuss.

Omar's Solution

$$
\begin{array}{r}
X - 1 \\
X + 4 \enclose{longdiv}{X^2 + 3X - 4} \\
\ominus \quad \ominus \\
X^2 + 4X \\
- X - 4 \\
\oplus \quad \oplus \\
\underline{- X - 4} \\
0 \quad\quad 0
\end{array}
$$

Khaled's Solution

$$
\begin{array}{cc}
X^2 & -4 \\
X & 4 \\
X & -1
\end{array}
$$

▶ **Find the quotient of each of the following expressions:**

⑦ $(X^2 + 9X + 20)$ divided by $(X + 4)$

⑧ $(X + X^2 - 2)$ divided by $(X - 1)$

⑨ $(2 + 2y^2 - 5y)$ divided by $(y - 2)$

⑩ $(X^2 - 10X + 25)$ divided by $(X - 5)$

⑪ $(7X - 5X^2 + 2X^3 - 6)$ divided by $(2X - 3)$

⑫ $(X^3 - 27)$ divided by $(X - 3)$

⑬ $(25y^2 - 9X^2)$ divided by $(5y - 3X)$

⑭ $(X^4 - 1)$ divided by $(X^2 + 1)$

▶ **Answer the following questions :**

⑮ Divide $(-3X^2 + X^3 - X + 6)$ by $(X - 2)$, then find the numerical value of the quotient when $X = 3$.

⑯ If $(X - 4)$ is a factor of the expression $(X^2 - 5X + 4)$, find the other factor.

⑰ Find the value of b that makes the expression $(4X^2 + 11X + b)$ divisible by $(4X - 1)$.

(18) **Floor Tiling:** A worker is tiling the floor of a rectangular room. The total area of the floor is equal to ($x^2 + 7x + 12$) square meters. If the width of the floor is ($x + 3$) meters, find the length of the floor in terms of x.

(19) **Manufacturing:** A company is packaging its products in boxes, each box in the shape of a rectangular prism with a volume of ($x^3 + 6x^2 + 11x + 6$) cubic centimeters, and a height of ($x + 2$) cm.
Find the area of the base of the box in terms of x.

Choosing appropriate adhesive materials is an essential step for the success of the tiling process

(20) **Science:** A car is moving in a straight line at a speed of ($2x + 4$) meters/hour. Calculate the time in hours that this car takes to cover a distance of ($2x^3 + 8x^2 + 8x$) meters in terms of x.

Adhering to the specified speed limits while driving helps avoid road accidents.

(21) **Decoration:** If the total cost of painting a wall is ($2x^2 + 8x + 6$) pounds, and the cost of painting one square meter of the wall is ($2x + 2$) pounds, find the area of the wall in terms of x.

The aesthetic effect is achieved through the selection of an appropriate paint colour.

(22) **Road Paving:** A construction company is paving a rectangular road. If the total area of asphalt required for the road is ($3x^3 + 14x^2 + 13x + 20$) square meters, and the width of the road is ($x + 4$) meters, find the length of the road in terms of x.

Road paving helps improve traffic safety.

Creative Thinking

(23) **Geometry:** ABC is a triangular piece of land of area of ($6x^2 + 7xy + 2y^2$) square meters. If the length of $\overline{AB}$ is ($2x + y$) meters, find the length of the perpendicular from C to $\overleftrightarrow{AB}$ in terms of x and y, and then find the numerical value of the length of this perpendicular when $x = 2$ and $y = 3$.

Evaluate *your understanding!*

How well do you understand dividing algebraic expressions? Tick the right box.

Unit Two Assessment
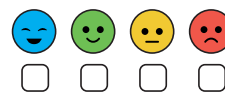
▶ **Choose the correct answer from the given ones:**

(1) If $-2X^3 \times X = a X^n$, what is the value of $a + n$?

(a) -3 (b) -2 (c) 2 (d) 3

(2) If $X \in Z$, which of the following is a solution to the inequality $1 - 2X < 3$?

(a) 0 (b) -1 (c) -2 (d) -4

(3) If $(X - 2)(X + 2) - 5 = 0$, what is the value of X where $X < 0$?

(a) -9 (b) -2

(c) -1 (d) -3

(4) A rectangle has an area of $(X^2 + 6X + 8)$ square units and a length of $(X + 4)$ length units; what is the width of the rectangle?

(a) X (b) $X + 2$

(c) $X - 2$ (d) $X - 4$

▶ **Complete the following:**

(5) $\dfrac{a^2 - a}{a - 1} = \dots\dots\dots$

(6) The coefficient of ab in the product $(2a - 3b) \times (a - b)$ equals $\dots\dots\dots$

(7) The solution set of the inequality $2X - 1 > X + 2$ in Z equals $\dots\dots\dots$

(8) If $(3X - 4)^2 = a X^2 + b X + c$, then $b = \dots\dots\dots$

▶ **Answer the following questions:**

(9) Find in simplest form the product $(X - 3)(2X^2 - X + 4)$ and then find the numerical value of the result when $X = -1$.

(10) Find the solution set of the inequality $2(3X - 1) \geq 4X - 3$ in Q.

(11) Find the quotient of $(-2X^2 y + 4X y^2 - 6X y)$ divided by $(-2X y)$.

(12) If the length of each side of a square is increased by 3 cm, its area increases by 51 square centimeters, what was the length of a side of the square before the increase?

Unit Two **Activity**

▶ **Activity Aim:**

Online research to develop collaborative learning skills using digital technology to understand one method of multiplying algebraic expressions called (FOIL Method).

▶ **Implementation Steps:**

1 Form working groups consisting of $(2 - 4)$ students to begin the activity.

2 Identify what the letters in the word (FOIL) refer to in multiplying algebraic expressions.

3 Describe how this method and the method presented in the student's book are related.

4 Collect data on the most significant mathematicians who have contributed and developed the concept of algebraic terms and expressions.

UNIT **3**

Geometry & Measurement
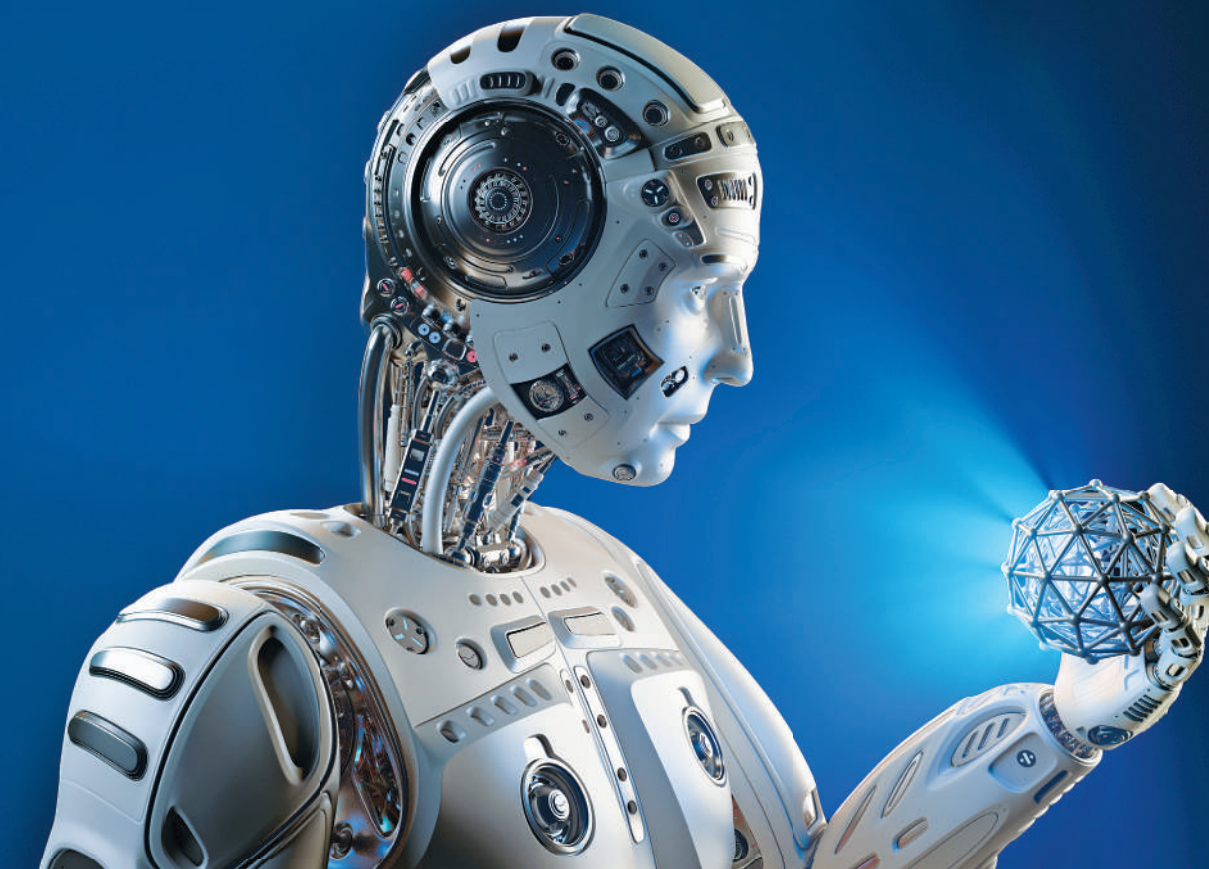
Scientists consider continuous learning essential to keep pace with developments in the effective use of artificial intelligence in the design and production of complex geometrical shapes.

▶ **Is it possible to use artificial intelligence (AI) to design complex geometrical shapes quickly and accurately?**

Issues and Life Skills
- Mathematical Communication
- Sustainable Development
- Information Technology
- Globalization
- Mathematical Understanding

Values
- Security and Safety
- Responsibility
- Respecting Others
- Belonging

■ Learning Outcomes

• Find the area of a rhombus given the lengths of its diagonals.
• Find the area of a square given the length of its diagonal.
• Find the area of a trapezium.
• Use the laws of area to solve problems.

■ Vocabulary

• Area
• Perimeter
• Rhombus
• Square
• Trapezium

■ Notation in Mathematics

Area is usually denoted by the letter A, and perimeter by the letter P, the length of a side of the square or rhombus is denoted by the letter s, the length of a rectangle by the letter ℓ, its width by the letter w, the height by the letter h, and the length of the base by the letter b.

Get Ready!

Aswan Governorate is located in the south of the Arab Republic of Egypt and is one of the governorates of Upper Egypt.

Aswan boasts numerous tourist attractions such as the Temple of Abu Simbel and the Philae Temples, and is distinguished by its stunning natural scenery along the Nile River, as well as the High Dam.

On the map, Aswan Governorate resembles a trapezium with dimensions as shown in the given map.

What is the approximate area of Aswan Governorate in square kilometers?

In this lesson, you will learn how to find the areas of certain geometric shapes such as the square, rhombus, and trapezium, which will help you solve such problems.

Think & Discuss!

The teacher provided the opposite triangles to his students in one of the classes to create different geometric shapes. One of them made a square, another made a parallelogram, and a third made a triangle.
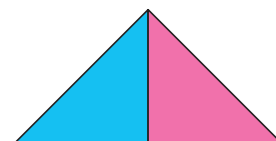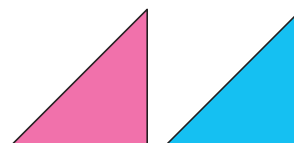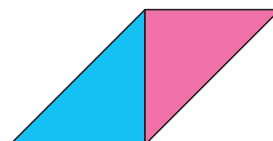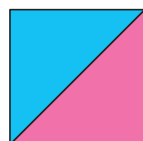
Do these shapes have the same area or the same perimeter? Discuss.

Learn!

In previous years, you studied the mathematical formulas for finding the areas and perimeters of certain geometric shapes such as:

Square	**Rectangle**	**Parallelogram**
$P = 4\,s$ $A = s^2$	$P = 2\,(\ell + w)$ $A = \ell \times w$	$P = 2\,(b_1 + b_2)$ $A = b_1 \times h$
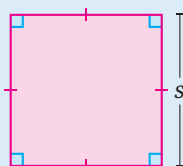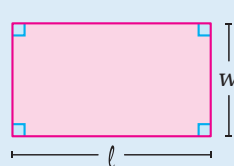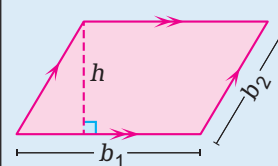
Rhombus	Triangle
$P = 4\,s$ $A = s \times h$	$P = a + b + c$ $A = \dfrac{1}{2}\,b \times h$

▶ In this lesson, you will learn more formulas to calculate the areas of geometric shapes.

Area of the Rhombus Given the Lengths of its Diagonals

Area of the rhombus $= \dfrac{1}{2}$ Product of the lengths of its diagonals

Let the area be A, and the lengths of the diagonals be d_1 and d_2:

$$A = \dfrac{1}{2} \times d_1 \times d_2$$
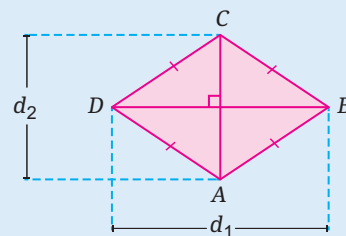
Example 1

For a rhombus with diagonal lengths of 5 meters and 8 meters, find its area.

✔

Area of the rhombus $= \dfrac{1}{2}$ Product of the lengths of its diagonals

$A = \dfrac{1}{2} \times d_1 \times d_2 = \dfrac{1}{2} \times 5 \times 8 = 20$

Thus, the area of the rhombus = 20 square meters

Example 2

For a rhombus with a side length of 10 feet, a height of 9.6 feet, and one diagonal length of 12 feet, find the length of the other diagonal.

✔

Area of the rhombus $=$ Side length $\times$ Height $= 10 \times 9.6 = 96$ square feet

$\because A = \dfrac{1}{2} \times d_1 \times d_2$

$\therefore 96 = \dfrac{1}{2} \times 12 \times d_2 \qquad \therefore 96 = 6 \times d_2 \qquad \therefore d_2 = 16$

Thus, the length of the other diagonal = 16 feet

The law of the area of a square can be deduced from the length of its diagonal as follows:

▶ Draw a square as follows:
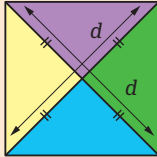
▶ Cut the square and then reattach it in this manner:
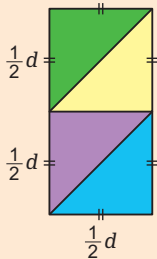
∴ The area of the square is equal to the area of the rectangle

$A = \frac{1}{2} d \times d = \frac{1}{2} d^2$.

■ **Self-Evaluation** ③

A square with a diagonal of 8 feet, and a parallelogram with a base length of 10 feet and the corresponding height of 4 feet.

Find the sum of their areas.

Area of the Square Given the Length of its Diagonal

A square is a rhombus with diagonals equal in length; thus:

The area of the square $= \frac{1}{2} \times$ length of the diagonal

$\times$ length of the diagonal

hence,

> the area of the square $= \frac{1}{2}$ square of the length of its diagonal
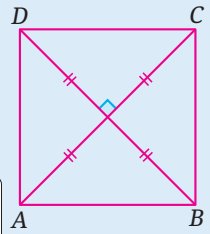
Assuming the area of the square A, and the length of the diagonal d,

So: $A = \frac{1}{2} d^2$

For example: the square with a diagonal length of 6 cm has an area in square centimeters:

$$A = \frac{1}{2} \times 6^2 = \frac{1}{2} \times 36 = 18$$

Example 3

Which has a greater area?

A square with a diagonal of 12 cm or a rectangle with a length of 11 cm and a width of 7 cm

✓

Assuming the area of the square A_1

$\therefore A_1 = \frac{1}{2} d^2 = \frac{1}{2} \times 12^2 = \frac{1}{2} \times 144 = 72$

Thus, the area of the square $= 72$ square centimeters

Assuming the area of the rectangle A_2

$\therefore A_2 = \ell \times w = 11 \times 7 = 77$

Thus, the area of the rectangle $= 77$ square centimeters

$\therefore$ Area of the rectangle $>$ Area of the square

Area of the Trapezium

A trapezium is a quadrilateral in which only two sides are parallel; each of the parallel sides is called a "base", and each of the non-parallel sides is called a "leg".

For example: In the opposite figure $ABCD$ is a trapezium, $\overline{AB}$ is the longer base, $\overline{DC}$ is the shorter base, and both $\overline{BC}$ and $\overline{AD}$ are legs.
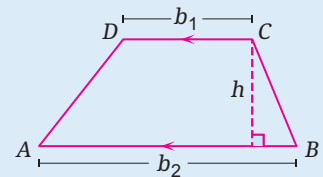
> The area of the trapezium
> $= \frac{1}{2}$ the sum of the lengths of the two parallel bases $\times$ the height

Assuming the area of the trapezium A, and the lengths of its two parallel bases b_1, b_2, and height h, So: $A = \frac{1}{2} (b_1 + b_2) \times h$

Note :

The middle base of the trapezium is
a line segment connecting the midpoints
of its legs.

Shorter base

Middle base

Longer base
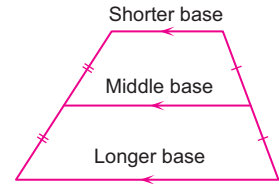
The length of the middle base
$= \frac{1}{2}$ the sum of the lengths of the two parallel bases.

Thus, the area of the trapezium = length of the middle base × height

Example 4

Calculate the area of the trapezium in each of the following shapes:

① 6 cm

6 cm

10 cm

② 5 feet

7.5 feet

15 feet

✅
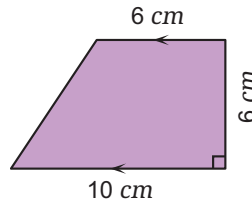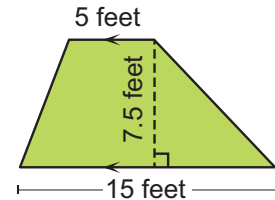
① $\because A = \frac{1}{2}(b_1 + b_2) \times h$

$\therefore A = \frac{1}{2}(6 + 10) \times 6 = 48$

Thus, the area of the trapezium = 48 square centimeters.

② $\because A = \frac{1}{2}(b_1 + b_2) \times h$

$\therefore A = \frac{1}{2}(5 + 15) \times 7.5 = 75$

Thus, the area of the trapezium = 75 square feet.

Example 5

**A trapezium has an area of 54 square centimeters and a height of 9 cm.
If the length of its shorter base is equal to 4 cm, find the length of its
longer base.**

✅

$\because A = \frac{1}{2}(b_1 + b_2) \times h$

$\therefore 54 = \frac{1}{2}(4 + b_2) \times 9$

$\therefore 4 + b_2 = 12 \qquad\qquad \therefore b_2 = 8$

Thus, the length of the longer base = 8 cm.

Lesson Assessment

First — Measuring Conceptual Understanding

▶ **Choose the correct answer from the given ones :**

(1) If the area of a rhombus is 100 square units, what is the product of the lengths of its diagonals?

(a) 25   (b) 50   (c) 100   (d) 200

(2) If the area of a square is 450 square units, what is the length of its diagonal in length units?

(a) 15   (b) 30   (c) 45   (d) 90

(3) A trapezium has the sum of the lengths of its two parallel bases equal to 16 cm, and its height is 5 cm. What is its area in square centimeters?

(a) 20   (b) 40   (c) 80   (d) 160

(4) In the opposite figure:

What is the length of $\overline{AB}$ in centimeters?

(a) 14   (b) 20

(c) 26   (d) 28

(5) A square has a side length of s and an area of A. What is the area of the square whose diagonal length is $2s$?

(a) A (b) $2A$ (c) $4A$ (d) A^2
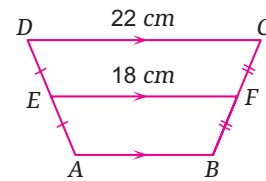
Second — Applying Scientific Concepts

▶ **Find the area of each of the following shapes:**

(6) 14.7 cm, 21.6 cm

(7) 7.6 m, 6.5 m, 12 m

(8) 6 feet, 6 feet

(9) A trapezium has an area of 175 square meters and the lengths of its two parallel bases are 14 meters and 21 meters. Find its height.

(10) Find the length of the diagonal of the square whose area is equal to the area of a rhombus with diagonal lengths of 4 meters and 25 meters.

(11) A trapezium has an area of 225 square inches, one of its parallel bases length is 23 inches, and its height is 7.5 inches. Find the length of its other base.

▶ **Find in terms of X the area of each of the following shapes, then find the numerical value of the area when $X = 4$:**

(12) $3X + 2$

(13) $5X$, $5X$, $3X + 3$, $3X + 3$

(14) A trapezium has an area of 315 square centimeters, a height of 15 cm, and the ratio between the lengths of its bases is 3:4. What is the length of each base?

⑮ Calculate the area of the trapezium *ABCD* if *AEFD*, *HIKG*, and *GKML* are three squares with equal side lengths

A 5 *cm* *D*

H *G* *L*
E *F*

B *I* *K* *M* *C*
├─────── 15 *cm* ───────┤
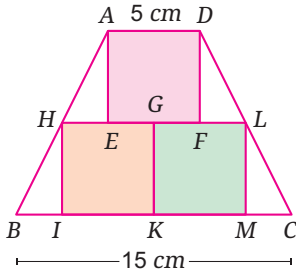
⑯ Calculate the area of the trapezium *ABEF* if *ABCD* is a square with a diagonal length of 6 *cm*.

E
F
5 *cm* 8 *cm*
A *D*
B *C*
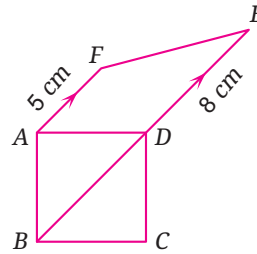
⑰ **Agriculture:** A square piece of agricultural land with a diagonal length of 8 kilometers, and its area equals the area of a rectangular farm with a width of 4 kilometers. Find the length of the farm.
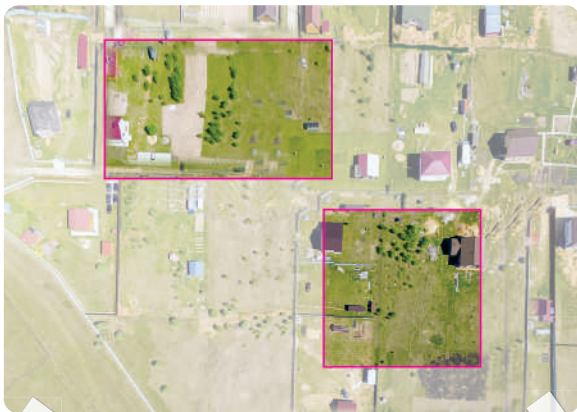
The agricultural sector in Egypt has witnessed a significant number of major agricultural projects, the most important of which is the one and a half million feddan project.

⑱ Two pieces of land are equal in area, the first in the shape of a rhombus with diagonal lengths of 8 meters and 27 meters, and the other in the shape of a trapezium its height is 6 meters. Find the length of its middle base.

⑲ **Decoration:** The following shape illustrates the dimensions of a room, and it is intended to tile its floor with ceramic tiles. If the cost of installing one square meter is 120 pounds, calculate the cost of installing the ceramic tiles.

├─ 4 *m* ─┤
2 *m*
4 *m* 4 *m*
├──── 6 *m* ────┤
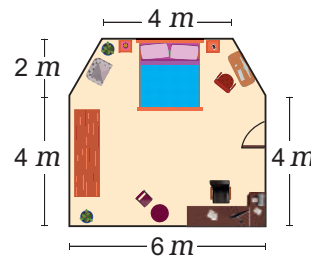
The distribution of furniture according to the area of the house reflects aesthetic awareness and refined taste.

Creative Thinking

⑳ Find the area of the coloured region in the following figure:

A
B *D*
2 *cm* 6 *cm* 2 *cm*
C
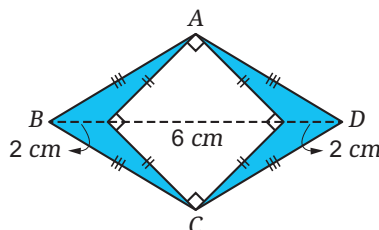
Evaluate *your understanding!*

How well do you understand areas?

Tick the right box

Geometrical Constructions

- Bisect an angle.
- Bisect a line segment.
- Draw a triangle given the lengths of its sides.
- Draw a triangle given the lengths of two sides and the measure of the included angle between them.
- Draw a triangle given the measures of two angles and the length of the side drawn between their vertices.

■ **Vocabulary**

- Geometrical Construction
- Angle
- Bisector
- Line Segment
- Triangle

■ **Note that**

The use of a compass and ruler in bisecting angles and line segments is one of the accurate methods of bisection, regardless of the measures of the angles and lengths of the line segments.

■ **Self-Evaluation** ①

Draw an angle of measure 45°, then bisect it using a ruler and compass. Verify by measuring that the bisection is accurate.

Get Ready!

When creating designs, engineers need to determine dimensions and angle measures accurately, using various geometrical tools.

An engineer discovered that the markings on his ruler were unclear; how can he determine the midpoint of a line segment in one of his designs?

In this lesson, you will learn how to use geometrical tools to perform some geometrical constructions, which will help you solve such real-life problems.

Think & Discuss!

You have an angle ABC of unknown measure, can you draw a ray that bisects this angle without measuring?
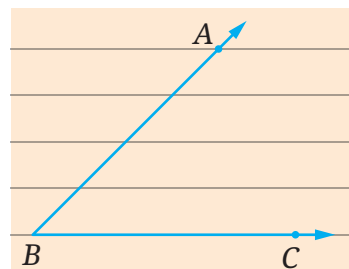
Learn!

1 | Bisecting an Angle

To bisect an angle such as $\angle B$, follow these steps:
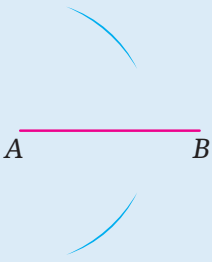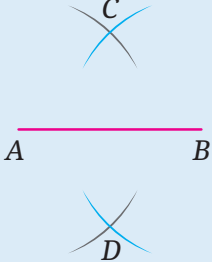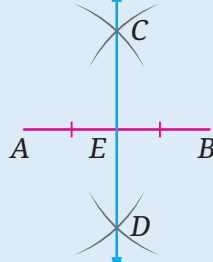
① Place the compass needle at vertex B, and draw an arc that intersects the sides of the angle at points A and C.

② With the same compass span, or any suitable span, place the compass needle at point A and draw an arc, then with the same span, place it at point C and draw another arc that intersects the first arc at point D.

③ Draw $\overrightarrow{BD}$, which will be the bisector of angle B, such that:
$m(\angle ABD)$
$= m(\angle CBD)$

To bisect a line segment such as $\overline{AB}$, follow these steps :

①

②

③

Set the compass span greater than half the length of $\overline{AB}$, then place the compass needle at A and draw two arcs in two different directions from $\overline{AB}$.

With the same compass span, place the compass needle at B and draw two arcs that intersect the previous arcs at points C and D.

Draw $\overleftrightarrow{CD}$ to intersect $\overline{AB}$ at E, thus point E is the midpoint of $\overline{AB}$, i.e.: $AE = EB$

3 | **Drawing a Triangle**
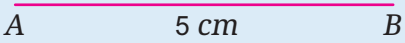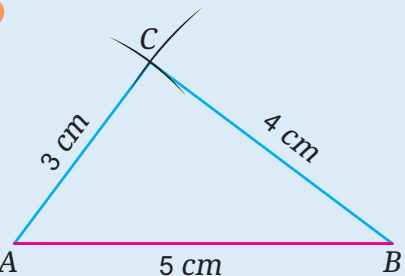
First | **Drawing a triangle given the lengths of its sides**

In the first term, you have learned how to draw a triangle given the lengths of its sides, let's remember how to do that :

To draw the triangle ABC where the length of $\overline{AB}$ equals 5 cm, the length of $\overline{BC}$ equals 4 cm, and the length of $\overline{AC}$ equals 3 cm, follow these steps :

①

A — 5 cm — B

Use a ruler to draw $\overline{AB}$ of length 5 cm.

②

A — 5 cm — B

Set the compass span to 4 cm. Place the compass needle at B and draw an arc.

③

A — 5 cm — B

Then set the compass span to 3 cm, place the compass needle at A and draw an arc that intersects the first arc at point C.

④

A — 5 cm — B

Draw $\overline{BC}$ and $\overline{AC}$ to get the triangle ABC with side lengths of 5 cm, 4 cm, and 3 cm.

Second — Drawing a triangle given the lengths of two sides and the measure of the included angle between them

To draw the triangle ABC where the length of $\overline{AB}$ is 4 cm and the length of $\overline{AC}$ is 3 cm, $m(\angle BAC) = 65°$, follow these steps :

1

A 4 cm B

Use a ruler to draw $\overline{AB}$ of length 4 cm.

2

65°
A 4 cm B

Using a protractor, from A, mark an angle of measure 65°, then draw a ray that defines the angle.

3

C
3 cm
65°
A 4 cm B

Set the compass span to 3 cm, then place its needle at A and draw an arc to intersect the drawn ray at C, thus the length of $\overline{AC}$ is equal to 3 cm.

4

C
3 cm
65°
A 4 cm B

Draw $\overline{BC}$ to get the triangle ABC where: $AB = 4$ cm, $AC = 3$ cm, $m(\angle A) = 65°$.

Third — Drawing a triangle given the measures of two angles and the length of the side drawn between their vertices

To draw the triangle ABC where the length of $\overline{AB}$ is 4 cm, $m(\angle A) = 50°$, $m(\angle B) = 45°$, follow these steps :

1

A 4 cm B

Draw $\overline{AB}$ of length 4 cm.

2

50°
A 4 cm B

From A, mark an angle of measure 50°, then draw a ray to define the angle.

3

From B, mark an angle of measure 45°, then draw a ray that defines this angle and intersects the first ray at C, thus you get the triangle ABC where :
$AB = 4$ cm, $m(\angle A) = 50°$, $m(\angle B) = 45°$.

C
50°   45°
A 4 cm B

■ **Self-Evaluation ④**

Draw the triangle ABC where :
$AB = AC = 5$ cm,
$m(\angle A) = 60°$
and determine, by measuring, the type of the triangle according to the lengths of its sides.

■ **Self-Evaluation ⑤**

Draw the triangle ABC where $AB = 5$ cm,
$m(\angle A) = 120°$,
$m(\angle B) = 30°$
and determine, by measuring, the type of the triangle according to the lengths of its sides.
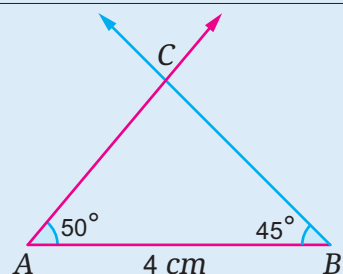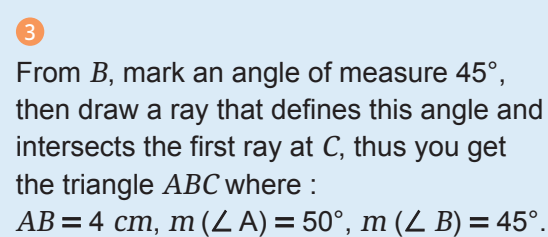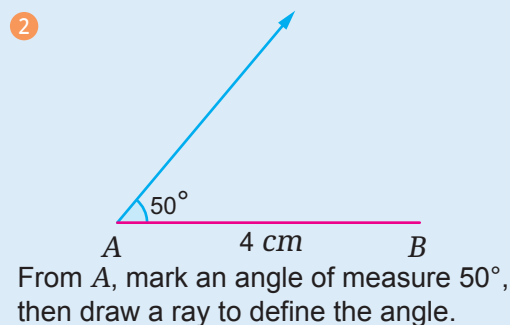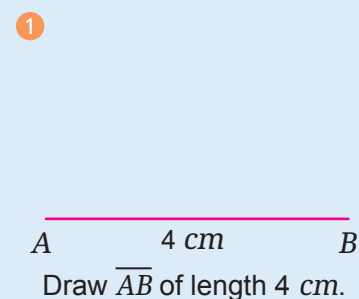
Draw the triangle XYZ where $XY = XZ = 6\ cm$, and $ZY = 8\ cm$.
Then bisect both $\angle Y$ and $\angle Z$ with bisectors intersecting at point M.
Verify by measuring that :
$MY = MZ$

Example

Using geometrical tools, draw the triangle ABC where the length of $\overline{AB}$ equals 4 cm, the length of $\overline{BC}$ equals 3 cm, and $m(\angle B) = 90°$.
Then bisect $\overline{AC}$ at point D.

Is $BD = \dfrac{1}{2} AC$?

✅
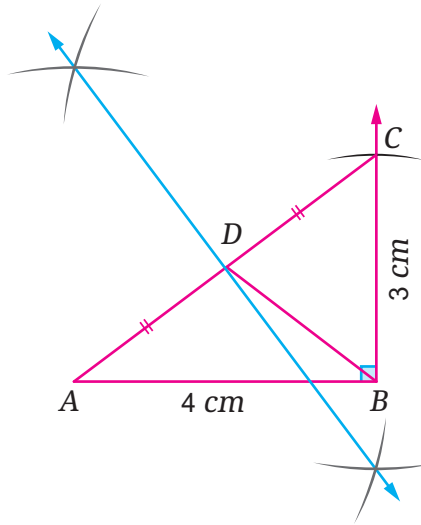
By measuring, you find that the length of $\overline{AC}$ equals 5 cm and the length of $\overline{BD}$ equals 2.5 cm,

thus: $BD = \dfrac{1}{2} AC$

Lesson Assessment

First | Measuring Conceptual Understanding

▶ **Choose the correct answer from the given ones:**

① When bisecting $\angle BAC$ with a compass, you find that:

  ① $m(\angle BAF) = $ ....

  (a) $m(\angle BFA)$
  (b) $m(\angle EAF)$
  (c) $m(\angle EFA)$
  (d) $m(\angle BAC)$
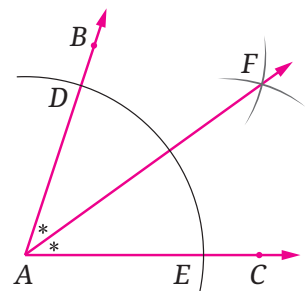
  ② The length of $\overline{EF}$ must equal the length of ....

  (a) $\overline{DF}$
  (b) $\overline{AD}$
  (c) $\overline{AE}$
  (d) $\overline{AF}$

② When bisecting the line segment $\overline{AB}$ with a compass, you must have ....

  (a) $AC < \dfrac{1}{2} AB$
  (b) $AC < AD$

  (c) $AC > \dfrac{1}{2} AB$
  (d) $AC < AE$
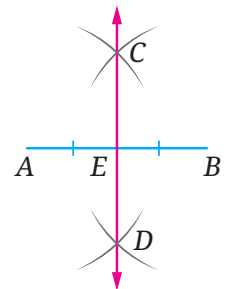
▶ **Use the geometrical tools in each of the following:**

③ Draw ∠ *ABC* of measure 120°, then bisect it using a ruler and compass by the bisector $\overrightarrow{BD}$, showing the steps of the solution. Verify by using a protractor that $m\,(\angle\,ABD) = m\,(\angle\,CBD)$.

④ Draw a line segment $\overline{AB}$ of length 7 *cm*, then bisect it using a ruler and compass at point *C*, showing the steps of the solution. Verify by using a ruler that *C* is the midpoint of $\overline{AB}$.

⑤ Draw △ *ABC* where the length of $\overline{AB}$ equals 7 *cm*, the length of $\overline{BC}$ equals 5 *cm*, and $m\,(\angle\,ABC) = 80°$, then determine by measuring the type of the triangle according to the measures of its angles.

⑥ Draw △ *ABC* where $m\,(\angle\,ABC) = 42°$, $m\,(\angle\,ACB) = 38°$, and the length of $\overline{BC}$ equals 6 cm, then determine by measuring the type of the triangle according to the lengths of its sides.

⑦ Draw △ *XYZ* where $XY = 6$ *cm*, $YZ = 4$ *cm*, $XZ = 5$ *cm*, then determine by measuring the type of the triangle according to the measures of its angles.

⑧ Draw the equilateral triangle *ABC*, the length of each side is 6 *cm*.

⑨ Draw △ *ABC* where $AB = 8$ *cm*, $m\,(\angle\,A) = 70°$, $m\,(\angle\,B) = 50°$, then bisect $\overline{AC}$ at point *D* and bisect $\overline{BC}$ at point *E*. Prove by measuring that: $AB = 2\,DE$.

⑩ Draw ∠ *ABC* of measure 60°, then bisect it using a ruler and compass by the bisector $\overrightarrow{BD}$, and then bisect both ∠ *ABD*, ∠ *CBD* by the bisectors $\overrightarrow{BE}$ and $\overrightarrow{BF}$ respectively. Prove by measuring that: $m\,(\angle\,ABF) = 3\,m\,(\angle\,CBF)$.

⑪ **Arts :** Use the geometrical tools to design a medal in the shape of an equilateral triangle, the length of each side is 4 *cm*, to be awarded to the student who wins in a Mathematics competition.

Creative Thinking

⑫ Use the geometrical tools to draw the bisectors of the interior angles of any triangle. What do you notice?

Participation in scientific competitions motivates you to learn and equips you with many skills in scientific research.

Evaluate *your understanding!*

How well do you understand geometrical constructions? Tick the right box

Get Ready!

The butterfly carries an exquisite artistic design on its wings that captivates us with its details.

If you closely observe the butterfly's wings, you will find that they are exactly identical.

Does one of the butterfly's wings appear to be a mirrored image of the other wing across a vertical line passing through the body of the butterfly?

Or does it appear as though it has been translated from one side to the other without rotating? Or does it appear as though it has rotated around a certain point?

In this lesson, you will learn some geometrical transformations, such as reflection, translation, and rotation, which will help you answer such questions.

Think & Discuss!

Why do you think the word "Ambulance" is written in reverse on the front of an ambulance? Discuss.

Learn!

Geometrical transformations

If all the points of a geometrical shape move according to a certain system, then you get an image of this shape in a new position. This shape is said to be under an effect of a geometrical transformation. Reflection, translation and rotation are types of geometrical transformations.

Reflection in a Line	Translation	Rotation
Reflection in a line is the creation of a mirrored image of the shape across a line known as the axis of reflection.	Translation is the displacement of the shape along a straight line a specified distance in a specified direction.	Rotation is the turning of the shape around a point known as the center of rotation at a specified angle in a specified direction.

The image resulting from reflection, translation, or rotation is congruent to the original shape.

| First | Reflection in the coordinate axes |

• The point and its image under reflection in a straight line are equidistant from that line, which is called the axis of reflection.
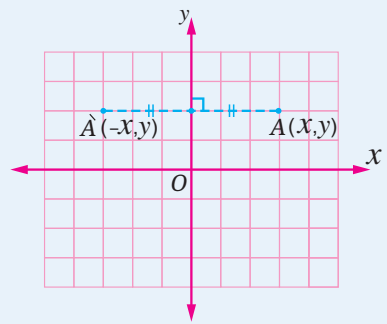
Reflection in the x-axis	Reflection in the y-axis

$$A (x , y) \xrightarrow[\text{in } x\text{-axis}]{\text{by reflection}} A` (x , -y)$$

$$A (x , y) \xrightarrow[\text{in } y\text{-axis}]{\text{by reflection}} A` (-x , y)$$
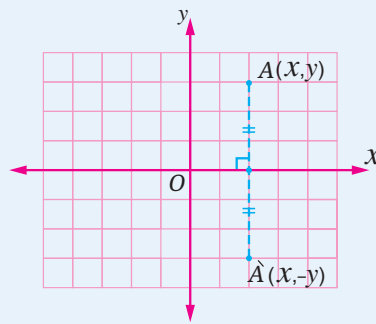
For example: The image of the point $(2 , 3)$ under reflection in the x-axis is the point $(2 , -3)$

For example: The image of the point $(-4 , 1)$ under reflection in the y-axis is the point $(4 , 1)$

Example 1

Draw the triangle whose vertices are the points $A (2, 0)$, $B (4, 1)$ and $C (1, 3)$, then draw its image under reflection in each of:

1 the x-axis.   **2** the y-axis.

✅ **1** **Under reflection in the x-axis:**

$$A (2 , 0) \xrightarrow[\text{in } x\text{-axis}]{\text{by reflection}} A (2 , 0)$$

$$B (4 , 1) \xrightarrow[\text{in } x\text{-axis}]{\text{by reflection}} B` (4 , -1)$$

$$C (1 , 3) \xrightarrow[\text{in } x\text{-axis}]{\text{by reflection}} C` (1 , -3)$$
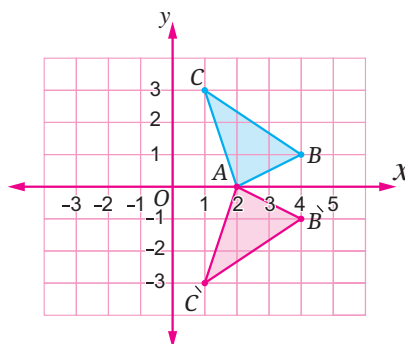
$\Delta AB`C`$ is the image of ΔABC under reflection in the x-axis.

2 **Under reflection in the y-axis:**

$$A (2 , 0) \xrightarrow[\text{in } y\text{-axis}]{\text{by reflection}} A` (-2 , 0)$$

$$B (4 , 1) \xrightarrow[\text{in } y\text{-axis}]{\text{by reflection}} B` (-4 , 1)$$

$$C (1 , 3) \xrightarrow[\text{in } y\text{-axis}]{\text{by reflection}} C` (-1 , 3)$$
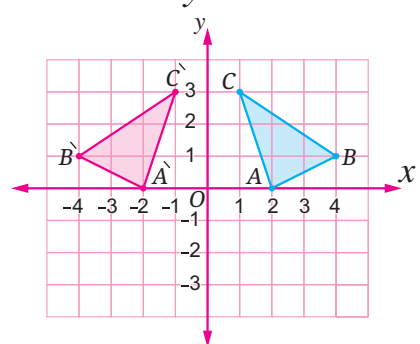
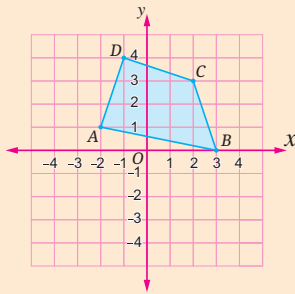$\Delta A`B`C`$ is the image of ΔABC under reflection in the y-axis.

Translation is
a transformation that
moves all points of a shape
the same distance in the
same direction.
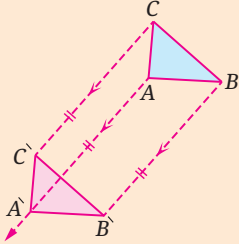
For example: The triangle
$A` B` C`$ is the image of the
triangle ABC by a translation
of distance $\overline{AA`}$ in the
direction of $\overrightarrow{AA`}$.

■ **Self-Evaluation** ②

Draw the image of the
quadrilateral $ABCD$ by
a translation $(2, -3)$.

■ **Note that**

In the ordered pair (a , b)
representing the translation:

▶ If a is positive, it signifies
a translation to the right,
and if it is negative, it
signifies a translation to
the left.

▶ If b is positive, it signifies
a translation upwards,
and if it is negative, it
signifies a translation
downwards.

Second | **Translation in the coordinate plane**

Translation in the coordinate plane is
determined by the horizontal displacement a
and the vertical displacement b, expressed
as the ordered pair (a , b), the image of the point
$A (x , y)$ by a translation of (a , b) is the point
$A` (x + a , y + b)$
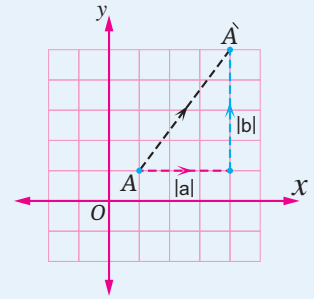
$$A (x , y) \xrightarrow[\;(a , b)\;]{\text{by translation}} A` (x + a , y + b)$$

For example: The image of the point $A (3 , 1)$ by a translation of 3 units
to the right and 4 units upwards, that is, a translation of $(3 , 4)$, is the
point $A` (3 + 3, 1 + 4)$, which is the point $A` (6 , 5)$.

Example ②

**Draw the triangle ABC with the vertices $A (-1 , 1)$, $B (3 , 1)$, $C (3 , 4)$,
then find its image by each of the following:**

❶ Translation of 3 units to the left. ❷ Translation $(1, -2)$

✔ ① The translation of 3 units to the
left is equivalent to a translation
of $(-3, 0)$.

$$A (-1 , 1) \xrightarrow[\;(-3 , 0)\;]{\text{by translation}} A` (-4 , 1)$$

$$B (3 , 1) \xrightarrow[\;(-3 , 0)\;]{\text{by translation}} B` (0 , 1)$$

$$C (3 , 4) \xrightarrow[\;(-3 , 0)\;]{\text{by translation}} C` (0 , 4)$$
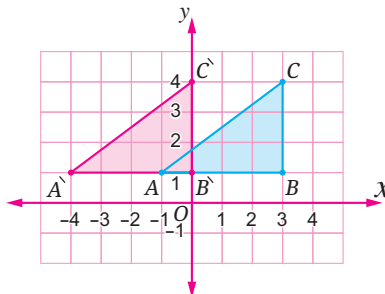
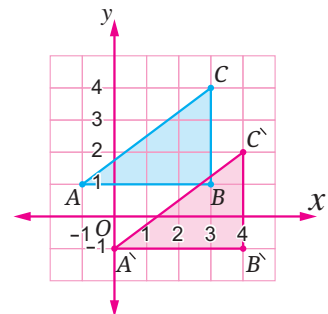

$\Delta A`B`C`$ is the image of ΔABC by
a translation of 3 units to the left.

② Translation $(1 , -2)$

$$A (-1 , 1) \xrightarrow[\;(1 , -2)\;]{\text{by translation}} A` (0 , -1)$$

$$B (3 , 1) \xrightarrow[\;(1 , -2)\;]{\text{by translation}} B` (4 , -1)$$

$$C (3 , 4) \xrightarrow[\;(1 , -2)\;]{\text{by translation}} C` (4 , 2)$$

$\Delta A`B`C`$ is the image of ΔABC
by a translation $(1 , -2)$

Rotation is denoted by the symbol:

$$R \qquad (O \qquad , \qquad \theta)$$

Rotation | Center of rotation | Measure of the angle of rotation and its direction
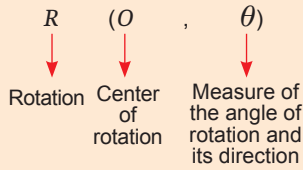
For example:

▶ Rotation around the origin in the anti-clockwise direction with an angle of measure 90° is written as $R(O, 90°)$

▶ Rotation around the origin in the clockwise direction with an angle of measure 90° is written as $R(O, -90°)$

Third | **Rotation in the coordinate plane**

• Rotation is a geometrical transformation that causes every point of the shape to rotate around a fixed point at a specified angle in a certain direction.

• The point around which the shape rotates is called the center of rotation, meaning that rotation is completely determined by the following elements:

1 Center of rotation.
2 Measure of the angle of rotation.
3 Direction of rotation.

• If the direction of rotation is:
 - Anti-clockwise, then θ is positive.
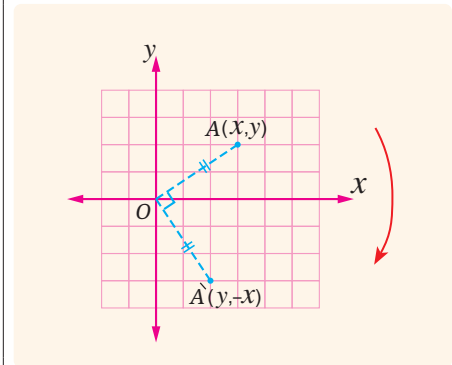 - Clockwise, then θ is negative.
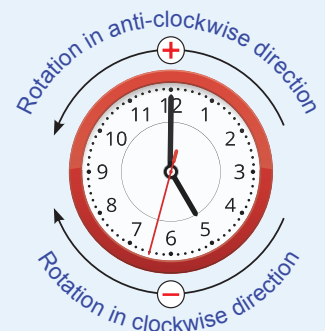
Rotation $R(O, 90°)$	Rotation $R(O, -90°)$
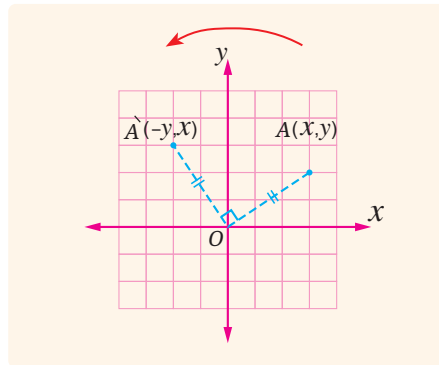
$$A(x, y) \xrightarrow{R(O, 90°)} A'(-y, x)$$

$$A(x, y) \xrightarrow{R(O, -90°)} A'(y, -x)$$

For example:

The image of the point (3 , 1) under rotation $R(O, 90°)$ is the point (– 1 , 3)

For example:

The image of the point (– 3 , 4) under rotation $R(O, -90°)$ is the point (4 , 3)

Think & Discuss!

What does happen to the shape when it rotates around the origin by an angle of measure ± 360°?

Rotation $R(O, ±180°)$

$$A(x, y) \xrightarrow{R(O, ±180°)} A'(-x, -y)$$

For example:

The image of the point (– 1 , 3) under rotation $R(O, ±180°)$ is the point (1 , – 3)

Technology

Use the GeoGebra programme to draw a triangle in the coordinate plane, then draw its image:

1 by reflection in the x-axis
2 by reflection in the y-axis
3 by translation (– 3 , 4)
4 by rotation $R(O, 90°)$
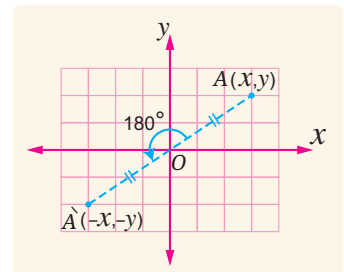
Note that

▶ Rotation $R(O, 90°)$ is equivalent to rotation $R(O, -270°)$
▶ Rotation $R(O, 270°)$ is equivalent to rotation $R(O, -90°)$

The square $ABCD$ has all its vertices located in the second quadrant.

If $A(-1, 1)$, $B(-1, 4)$, draw the square $ABCD$ in the coordinate plane and then find its image under each of the following:

① $R(O, -180°)$

② $R(O, 270°)$

A shape possesses rotational symmetry around its center if it can be rotated by an angle of measure less than 360° around its center and remain the same shape as it was in its original position.

▸ A starfish is a remarkable example of an animal that exhibits rotational symmetry. The "ideal" starfish has a rotational symmetry of 72 degrees.

If you place a point at the center of the starfish, you can rotate it around that point by any multiple of 72 degrees, and it will appear exactly the same!

Example 3

Draw the triangle ABC in the coordinate plane where $A(-1, 2)$, $B(3, 1)$, $C(0, 4)$, and then draw its image under each of the following rotations:

① $R(O, 90°)$ **②** $R(O, -90°)$ **③** $R(O, 180°)$

✅

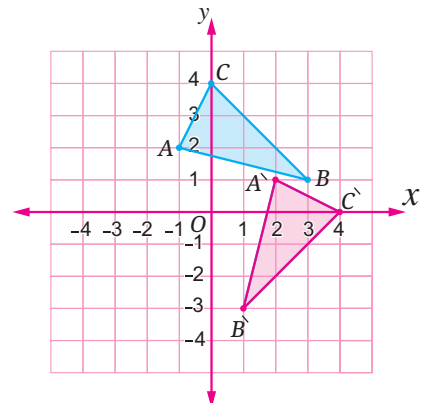**①**

$A(-1, 2) \xrightarrow{R(O, 90°)} A'(-2, -1)$

$B(3, 1) \xrightarrow{R(O, 90°)} B'(-1, 3)$

$C(0, 4) \xrightarrow{R(O, 90°)} C'(-4, 0)$

$\Delta A'B'C'$ is the image of ΔABC under rotation $R(O, 90°)$
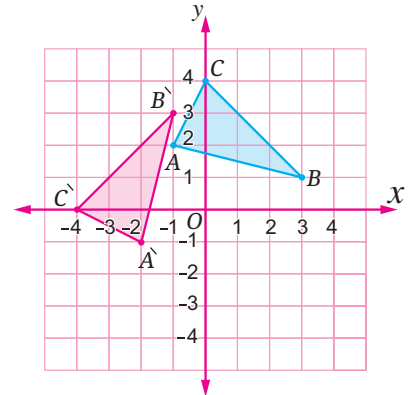
②

$A(-1, 2) \xrightarrow{R(O, -90°)} A'(2, 1)$

$B(3, 1) \xrightarrow{R(O, -90°)} B'(1, -3)$

$C(0, 4) \xrightarrow{R(O, -90°)} C'(4, 0)$

$\Delta A'B'C'$ is the image of ΔABC under rotation $R(O, -90°)$

③

$A(-1, 2) \xrightarrow{R(O, 180°)} A'(1, -2)$

$B(3, 1) \xrightarrow{R(O, 180°)} B'(-3, -1)$

$C(0, 4) \xrightarrow{R(O, 180°)} C'(0, -4)$

$\Delta A'B'C'$ is the image of ΔABC under rotation $R(O, 180°)$
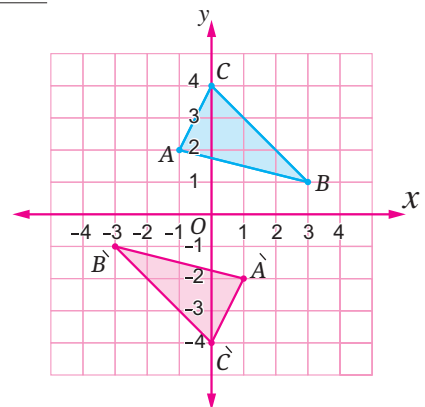
Properties of Reflection, Translation, and Rotation

▸ Both reflection in a straight line, translation, and rotation around a point in the plane preserve:

① Lengths of line segments. **②** Measures of angles.

③ Parallelism. **④** Betweeness.

**First** **Measuring Conceptual Understanding**

▶ **Choose the correct answer from the given ones:**

① What is the image of the point $(-1, 3)$ by reflection in the X-axis?

(a) $(1, 3)$ (b) $(-1, -3)$

(c) $(1, -3)$ (d) $(3, -1)$

② What is the image of the point $(5, 0)$ by reflection in the Y-axis?

(a) $(5, 0)$ (b) $(-5, 0)$

(c) $(0, 5)$ (d) $(0, -5)$

③ What is the image of the point $(0, -3)$ by a translation $(-1, 2)$?

(a) $(-1, -1)$ (b) $(-1, 1)$

(c) $(1, -1)$ (d) $(1, 1)$

④ What is the image of the point $(-4, 2)$ when rotated around the origin O by an angle of measure 90° anti-clockwise?

(a) $(-4, -2)$ (b) $(4, 2)$

(c) $(-2, 4)$ (d) $(-2, -4)$

⑤ Which of the following rotations makes the point $A'(x, -y)$ the image of $A(-x, y)$?

(a) $R(O, -90°)$ (b) $R(O, 90°)$

(c) $R(O, 180°)$ (d) $R(O, 360°)$

⑥ If A' is the image of the point A by reflection in the X-axis, and the point A is located in the third quadrant, in which quadrant is the point A' located?

(a) First   (b) Second

(c) Third   (d) Fourth

⑦ What is the image of the point $(5, -2)$ by a translation of 5 units in the negative direction of the X-axis?

(a) $(5, -7)$ (b) $(10, -2)$ (c) $(0, -2)$ (d) $(5, -3)$

⑧ What rotation makes the image of the point $A(2, -6)$ become $A'(-6, -2)$?

(a) $R(O, -180°)$ (b) $R(O, -90°)$ (c) $R(O, 90°)$ (d) $R(O, 180°)$

⑨ Which of the following represents the rotation of the opposite square around its center by an angle of measure 90° in the clockwise direction?
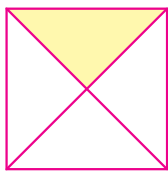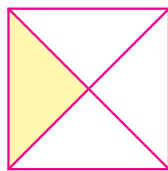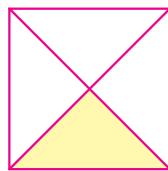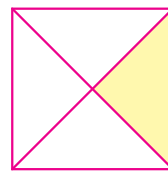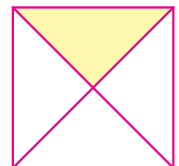

(a)   (b)   (c)   (d)

▶ **Choose the correct answer from the given ones:**

(10) What is the image of the point (2 , − 1) by the translation $(x , y) \longrightarrow (x − 3 , y + 4)$?
(a) (− 1 , 5)　　　　(b) (− 3 , 4)
(c) (5 , 3)　　　　(d) (− 1 , 3)

(11) If the point $A^{\backslash}$ (2 , 5) is the image of the point A by the translation $(x, y) \longrightarrow (x , y − 2)$, then the point A is ·········
(a) (2 , 3)　　　　(b) (2 , 7)
(c) (0 , − 2)　　　　(d) (0 , 7)

(12) What is the translation that makes the point $A^{\backslash}$ (− 2 , 1) the image of the point A (4 , − 5)?
(a) (− 6 , 6)　　　　(b) (− 6 , − 4)
(c) (2 , − 4)　　　　(d) (6 , − 6)

(13) If the point $A^{\backslash}$ $(x + 1 , − 2)$ is the image of the point A (− 4 , 2) by rotation around the origin O by an angle of measure 180°, what is the value of x?
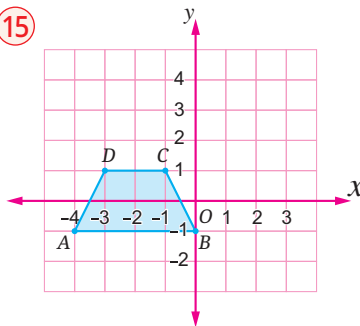(a) 3　　(b) − 1　　(c) − 2　　(d) − 5

▶ **Draw the image of each of the following figures using the geometrical transformation mentioned below the figure:**
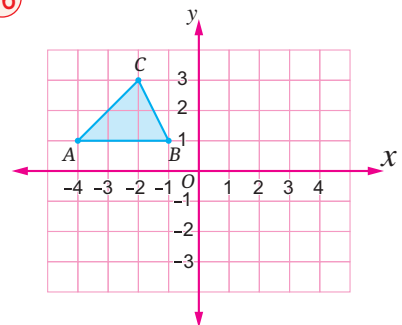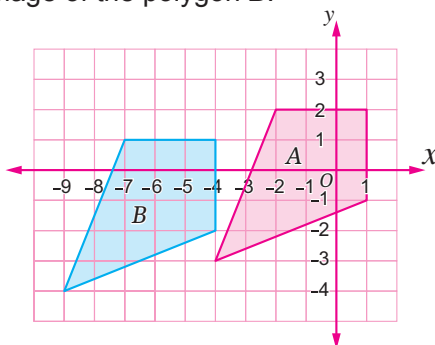
(14)
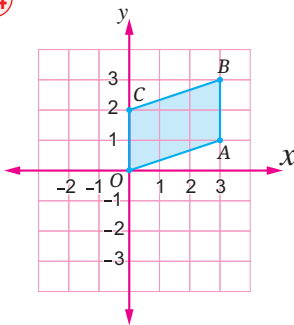
Reflection in the x-axis
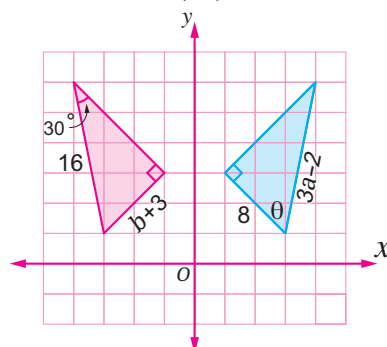
(15)

Translation (3 , 4)

(16)

Rotation around the origin O in the clockwise direction by an angle of measure 90°

(17) In the following figure, determine the translation that makes the polygon A the image of the polygon B.

(18) In the following figure, if one triangle is the image of the other by reflection in the y-axis, find the values of a, b, and θ.


▶ **In each of the following, the point and its image by reflection in one of the coordinate axes, determine whether the reflection is in the x-axis or the y-axis:**

(19) F (5 , − 6) $\longrightarrow$ $F^{\backslash}$ (− 5 , − 6)

(20) M (7 , − 9) $\longrightarrow$ $M^{\backslash}$ (7 , 9)

(21) N (− 3 , 0) $\longrightarrow$ $N^{\backslash}$ (− 3 , 0)

(22) C (0 , 5) $\longrightarrow$ $C^{\backslash}$ (0 , 5)

㉓ Draw $\triangle ABC$ where $A(-6, 6)$, $B(-2, 2)$, $C(4, 1)$, then draw its image under each of the following transformations:

① Rotation $R(O, -90°)$. ② Rotation $R(O, 90°)$.

③ Reflection in the y-axis. ④ Translation $(x, y) \longrightarrow (x-1, y+3)$.

㉔ Find the image of the polygon $ABCD$ where $A(4, -1)$, $B(6, -3)$, $C(9, -4)$, $D(7, 1)$, under rotation $R(O, -270°)$.

Third | **Analysis and Subjects Integration**

㉕ **Animation:** Various geometrical transformations are used in the production of animations. The opposite graph represents the movement of one shape in different positions with the coordinates of the position.
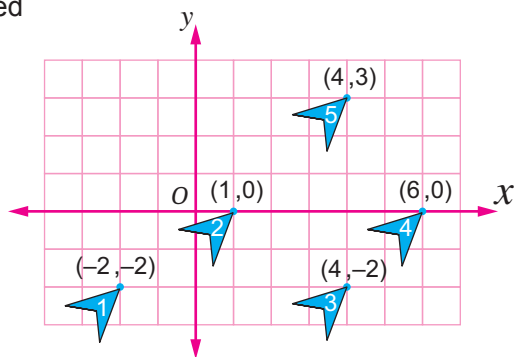
① Find the translation that makes shape 5 the image of shape 1.

② Find the translation that makes shape 4 the image of shape 1.

③ Find the translation that makes shape 3 the image of shape 2.

㉖ **Entertainment:**

The Ferris wheel opposite carries 20 carriages.

① What is the measure of the angle of rotation if seat 1 is moved to the position of seat 5?

② If seat 1 rotates by an angle of measure 144°, around the center of the wheel what number is the seat now occupying?
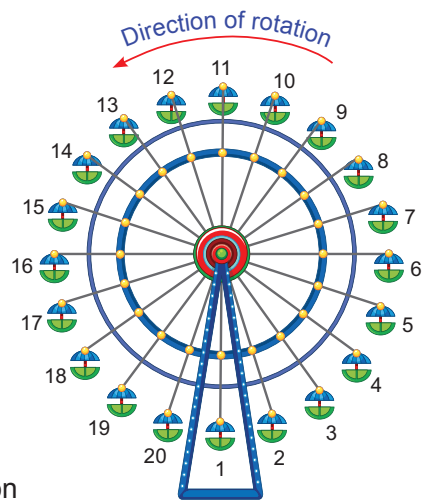
Creative Thinking

㉗ In the following figure, if the image of the number 25 by reflection in the y-axis is A, and its image by reflection in the x-axis is B, what is the value of $\sqrt[3]{B-A}$?
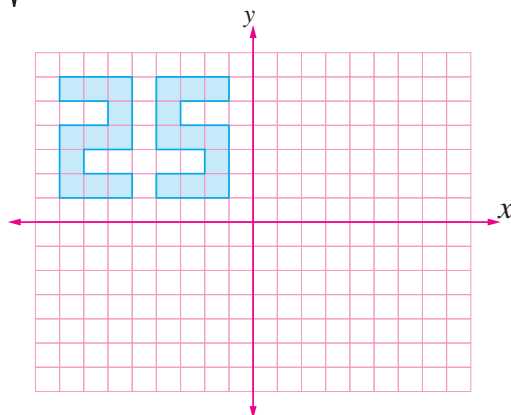
Composite of Geometrical Transformations

Get Ready!

Mathematics blends with arts and designs, and geometric patterns have been used in various forms in Islamic art and architecture.

In the picture, there is one of the beautiful designs from Islamic architecture; can you identify the geometrical transformations used in creating this design?

In this lesson, you will learn how to perform simple compositions of geometrical transformations (reflection, translation, and rotation), which will help you answer such questions.

Think & Discuss!

In the opposite picture, what transformation or transformations should be applied to make the bird B an image of the bird A?
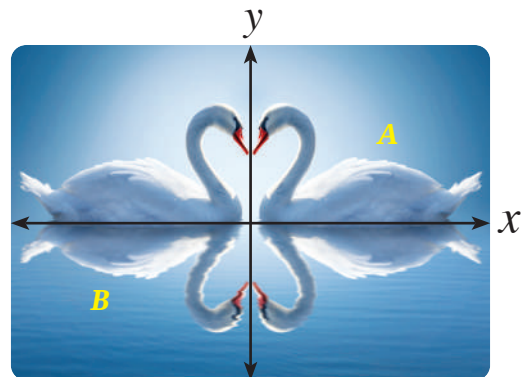
Learn!

Composite geometrical transformations

It is a series of successive geometrical transformations applied to a geometric shape, and sometimes the resulting geometric shape from the composition can be described by a single equivalent geometrical transformation.

For example: The arrow C is the image of the arrow A by reflection in the y-axis followed by reflection in the x-axis.

Example 1

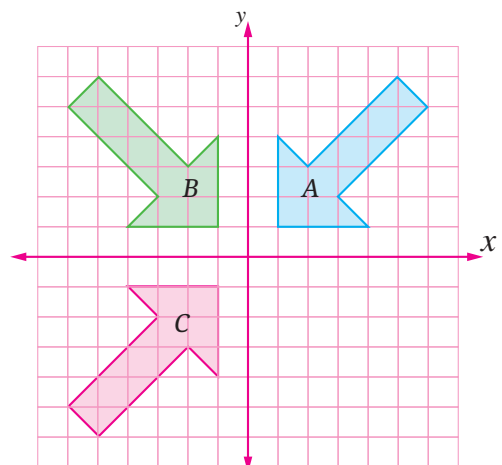**Draw $\triangle\,A^{\backslash}B^{\backslash}C^{\backslash}$ as the image of $\triangle\,ABC$ by reflection in the x-axis, then draw $\triangle\,A^{\backslash\backslash}B^{\backslash\backslash}C^{\backslash\backslash}$ as the image of $\triangle\,A^{\backslash}B^{\backslash}C^{\backslash}$ by reflection in the y-axis.**
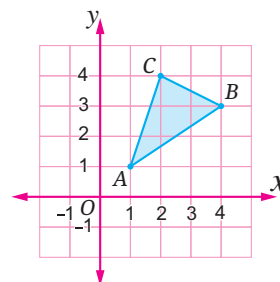
$A\,(1\,,\,1)\ \xrightarrow[\text{in }x\text{-axis}]{\text{by reflection}}\ A^{\backslash}\,(1\,,-1)\ \xrightarrow[\text{in }y\text{-axis}]{\text{by reflection}}\ A^{\backslash\backslash}\,(-1\,,-1)$

$B\,(4\,,\,3)\ \xrightarrow[\text{in }x\text{-axis}]{\text{by reflection}}\ B^{\backslash}\,(4\,,-3)\ \xrightarrow[\text{in }y\text{-axis}]{\text{by reflection}}\ B^{\backslash\backslash}\,(-4\,,-3)$

$C\,(2\,,\,4)\ \xrightarrow[\text{in }x\text{-axis}]{\text{by reflection}}\ C^{\backslash}\,(2\,,-4)\ \xrightarrow[\text{in }y\text{-axis}]{\text{by reflection}}\ C^{\backslash\backslash}\,(-2\,,-4)$

$\triangle\,A^{\backslash\backslash}B^{\backslash\backslash}C^{\backslash\backslash}$ is the image of $\triangle\,ABC$ by reflection in the x-axis followed by reflection in the y-axis.
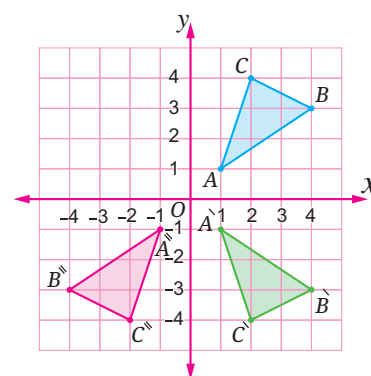
Example 2

Draw $\triangle\,ABC$ where $A\,(-2\,,-1)$, $B\,(3\,,\,0)$, $C\,(2\,,\,4)$, then draw its image by translation $(-2\,,\,1)$ followed by translation $(0\,,-5)$.

$A\,(-2\,,-1)\ \xrightarrow[(-2\,,\,1)]{\text{by translation}}\ A^{\backslash}\,(-4\,,\,0)\ \xrightarrow[(0\,,-5)]{\text{by translation}}\ A^{\backslash\backslash}\,(-4\,,-5)$

$B\,(3\,,\,0)\ \xrightarrow[(-2\,,\,1)]{\text{by translation}}\ B^{\backslash}\,(1\,,\,1)\ \xrightarrow[(0\,,-5)]{\text{by translation}}\ B^{\backslash\backslash}\,(1\,,-4)$

$C\,(2\,,\,4)\ \xrightarrow[(-2\,,\,1)]{\text{by translation}}\ C^{\backslash}\,(0\,,\,5)\ \xrightarrow[(0\,,-5)]{\text{by translation}}\ O\,(0\,,\,0)$

$\triangle\,A^{\backslash\backslash}B^{\backslash\backslash}O$ is the image of $\triangle\,ABC$ by translation $(-2\,,\,1)$ followed by translation $(0\,,-5)$
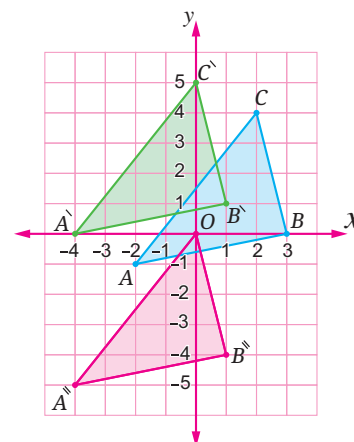
Copy the triangle ABC onto graph paper, then draw its image by rotation $R\,(O\,,\,90°)$ followed by rotation $R\,(O\,,\,90°)$.
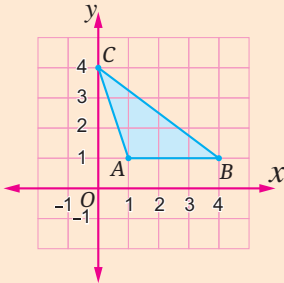
■ **Critical Thinking**

What geometrical transformation is equivalent to rotation $R\,(O\,,\,90°)$ followed by rotation $R\,(O\,,\,180°)$?

Example 3

Draw the image of the triangle ABC under rotation $R\,(O\,,\,90°)$ followed by rotation $R\,(O\,,\,180°)$.

✅

$$A\,(-4\,,\,1) \xrightarrow{R\,(O\,,\,90°)} A^{\backslash}\,(-1\,,\,-4) \xrightarrow{R\,(O\,,\,180°)} A^{\backslash\backslash}\,(1\,,\,4)$$

$$B\,(-1\,,\,1) \xrightarrow{R\,(O\,,\,90°)} B^{\backslash}\,(-1\,,\,-1) \xrightarrow{R\,(O\,,\,180°)} B^{\backslash\backslash}\,(1\,,\,1)$$

$$C\,(-2\,,\,3) \xrightarrow{R\,(O\,,\,90°)} C^{\backslash}\,(-3\,,\,-2) \xrightarrow{R\,(O\,,\,180°)} C^{\backslash\backslash}\,(3\,,\,2)$$

$\Delta\,A^{\backslash\backslash}B^{\backslash\backslash}C^{\backslash\backslash}$ is the image of $\Delta\,ABC$ under rotation $R\,(O\,,\,90°)$ followed by rotation $R\,(O\,,\,180°)$
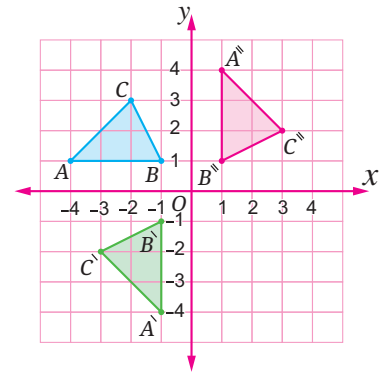
Lesson Assessment

First **Measuring Conceptual Understanding**

▶ **Choose the correct answer from the given ones:**

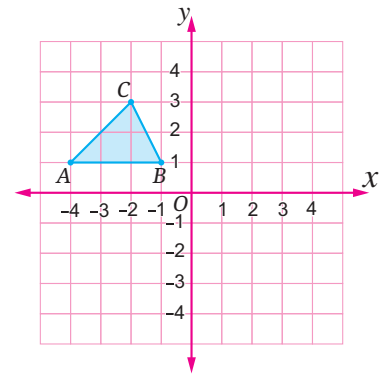① What geometrical transformation is equivalent to reflection in the x-axis followed by reflection in the y-axis?
(a) Rotation $R\,(O\,,\,90°)$ (b) Rotation $R\,(O\,,\,180°)$
(c) Rotation $R\,(O\,,\,360°)$ (d) Rotation $R\,(O\,,\,270°)$

② What geometrical transformation is equivalent to translation $(1\,,\,3)$ followed by translation $(0\,,\,2)$?
(a) Translation $(1\,,\,5)$ (b) Translation $(-1\,,\,-1)$
(c) Translation $(1\,,\,1)$ (d) Translation $(0\,,\,5)$

③ **In the opposite figure:**

What is the image of $\Delta\,DKO$ under reflection in the x-axis followed by reflection in the y-axis?
(a) $\Delta\,BFO$ (b) $\Delta\,CKO$
(c) $\Delta\,AFO$ (d) $\Delta\,BGO$
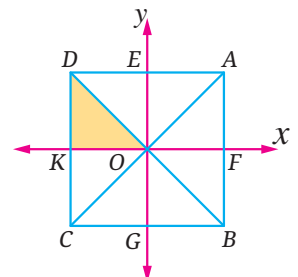
④ **In the opposite figure:**

Which of the following transformations makes the triangle B an image of the triangle A?

(a) A translation of 4 units to the right followed by a translation of 2 units upwards.
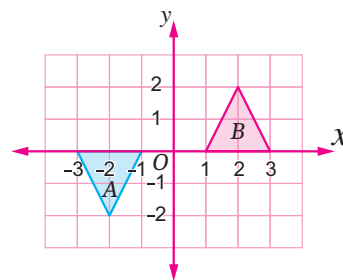
(b) A reflection in the y-axis, followed by a reflection in the x-axis.

(c) A rotation $R(O, 180°)$, followed by a rotation $R(O, 180°)$

(d) A rotation $R(O, 90°)$, followed by a rotation $R(O, 180°)$

Second **Applying Scientific Concepts**

▶ **Choose the correct answer from the given ones:**

⑤ What is the image of the point $(2, -3)$ by reflection in the x-axis followed by reflection in the y-axis?

(a) $(2, 3)$　　　　　(b) $(-2, -3)$

(c) $(-2, 3)$　　　　(d) $(3, 2)$

⑥ What is the image of the point $(-3, 5)$ by reflection in the x-axis followed by reflection in the x-axis again?

(a) $(3, -5)$　　　　(b) $(-3, -5)$

(c) $(-3, 5)$　　　　(d) $(3, 5)$

⑦ What is the image of the point $(-2, 4)$ under rotation $R(O, 90°)$ followed by rotation $R(O, -180°)$?

(a) $(-4, 2)$　　　　(b) $(4, 2)$

(c) $(-4, -2)$　　　(d) $(4, -2)$

⑧ What is the image of the point $(-1, 0)$ by translation $(1, 0)$ followed by translation $(2, -3)$?

(a) $(2, -3)$　　　　(b) $(0, 0)$

(c) $(1, 0)$　　　　(d) $(-1, 0)$

⑨ What is the image of the point $(-3, 0)$ by rotation $R(O, 90°)$ followed by rotation $R(O, -90°)$?

(a) $(3, 0)$　　　　(b) $(0, 3)$

(c) $(0, -3)$　　　(d) $(-3, 0)$

⑩ What is the image of the point $(-2, 3)$ by translation $(x, y) \longrightarrow (x+1, y-2)$ followed by translation $(-1, 2)$?

(a) $(-3, 5)$　　　　(b) $(0, 0)$

(c) $(-4, 3)$　　　　(d) $(-2, 3)$

▶ **Copy each figure onto a graph paper, then draw its image by the geometrical transformations mentioned below:**

⑪
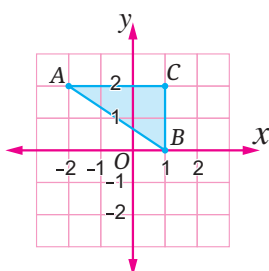
Reflection in the x-axis followed by reflection in the y-axis

⑫

Translation $(-1, 0)$ followed by translation $(-1, 5)$

⑬
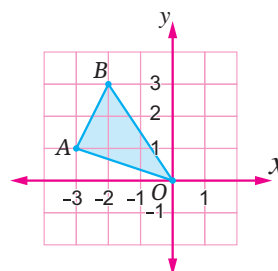
Rotation $R(O, 90°)$ followed by rotation $R(O, -180°)$

⑭ Draw the triangle ABC in the Cartesian plane where A (1 , 0), B (1 , 4), C (3 , 4). Then draw its image by reflection in the x-axis followed by reflection in the y-axis.
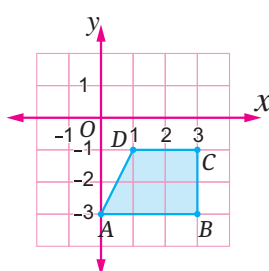
⑮ Draw $\overline{AB}$ in the Cartesian plane where A (− 1 , 0), B (0 , 3). Then draw the image of $\overline{AB}$ under rotation R (O , 90°) followed by rotation R (O , 180°).

⑯ Draw the square $ABCD$ in the Cartesian plane where A (− 1 , 0), B (3 , 0), C (3 , 4), D (− 1 , 4). Then draw its image by the translation (1 , 0) followed by the translation $(x , y) \longrightarrow (x − 2 , y)$.

▶ **Spot the mistake :**

⑰ Both Ahmed and Khadija found the image of the point A (3 , − 4) by the translation $(x , y) \longrightarrow (x + 2 , y + 5)$, followed by the translation (0 , − 5). Did both Ahmed and Khadija follow the correct method in their solutions? Discuss.

Ahmed's Solution :

$$A(3,-4) \xrightarrow[\ (2,\ 5)\ ]{By\ translation} A'(5,1) \xrightarrow[\ (0,-5)\ ]{By\ translation} A''(5,-4)$$

Khadija's Solution :

$$A(3,-4) \xrightarrow[\ (0,-5)\ ]{By\ translation} A'(3,-9) \xrightarrow[\ (2,\ 5)\ ]{By\ translation} A''(5,-4)$$

Creative Thinking

⑱ Using the geometrical transformation $(x , y) \longrightarrow (−y , x − 2)$, find the image of the triangle in the opposite figure, and state the composition used in this geometrical transformation.
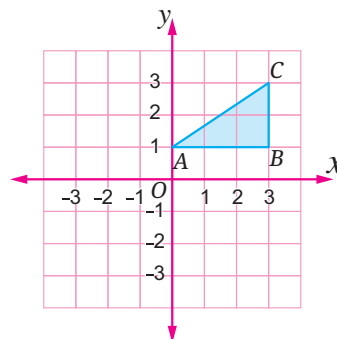
Unit Three Assessment
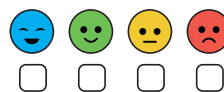
▶ **Choose the correct answer from the given ones:**

1 A square with a diagonal length of 12 feet, its area = ············ square feet.
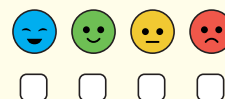
  (a) 36   (b) 72

  (c) 144   (d) 180

2 Which of the following is the image of the point $(-2, -5)$ by reflection in the y-axis?

  (a) $(2, 5)$ (b) $(2, -5)$

  (c) $(-2, 5)$ (d) $(-5, -2)$

3 A trapezium with a height of 3 cm and a middle base length of 10 cm, its area = ············ square centimeters.

  (a) 7.5   (b) 15   (c) 30   (d) 60

4 The rotation that makes the image of a shape is the shape itself is a rotation around the origin by an angle of measure ············

  (a) 90°   (b) $-90°$ (c) 180°   (d) 360°

▶ **Complete each of the following with the correct answer:**

5 A rhombus with a side length of 15 cm and diagonal lengths of 18 cm and 24 cm, its height = ············ cm.

6 The translation that makes the point $A^`(-3, 2)$ the image of the point $A(5, -3)$ is ············

7 A trapezium with a height of 5 cm and base lengths of 10 cm and 12 cm, its area = ············ square centimeters.

8 The image of the point $(4, 5)$ under the translation $(-2, 2)$ is ············

▶ **Answer the following questions:**

9 Draw an angle of measure 45° and then bisect it using a ruler and compass.

10 Draw the triangle ABC on a grid where $A(1, 2)$, $B(1, -1)$, $C(5, 2)$ and then draw its image under the rotation $R(O, -90°)$.

11 A trapezium with base lengths of 6 meters, X meters, and a height of 9 meters, has an area of 72 square meters. Find the value of X.

12 Draw $\overline{AB}$ on a grid where $A(2, 1)$, $B(3, 5)$ and then draw its image under reflection in the X-axis followed by reflection in the y-axis.

Unit Three Activity   Geometrical Constructions and Decoration

▶ **Activity Aim :** To deepen understanding and develop students' skills in geometrical constructions and apply their knowledge in life situations and in the decoration of a room.

▶ **Implementation Steps: In collaboration with two of your classmates:**

1 Identify the appropriate location to hang a picture on a wall with dimensions (4 meters, 3.5 meters) in a room so that the picture is in the center of the wall and at a height of (2 meters) from the floor.

2 Identify the suitable geometrical construction to determine the location of the nail at a height of (2 meters) from the floor and in the center of the wall.

3 Use an appropriate scale drawing to draw a rectangle representing the wall, then geometrically illustrate the steps to determine the position of the nail using the geometrical construction you have chosen.

4 State the steps followed to determine the position of the nail.

UNIT 4

Probabilities
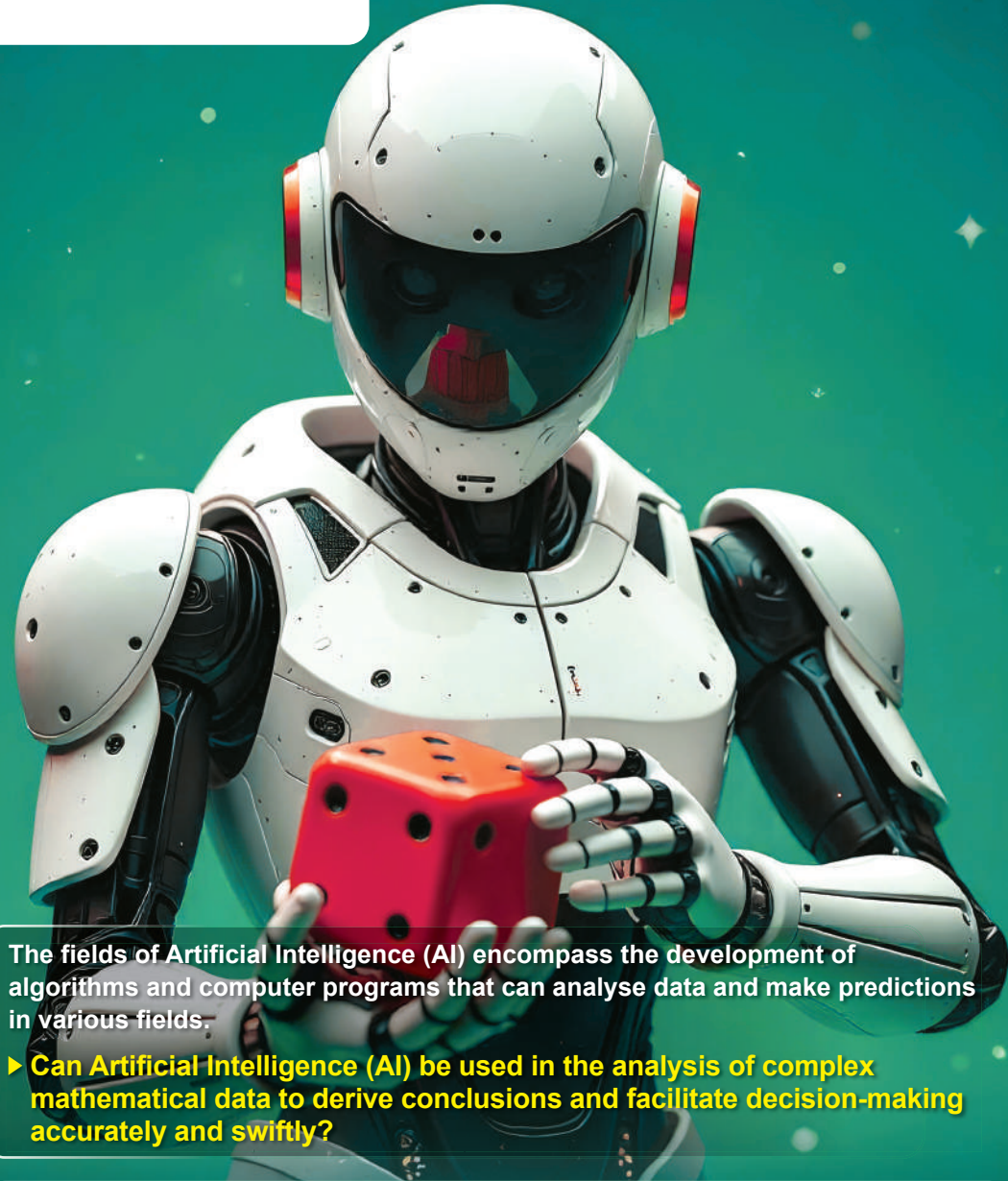
The fields of Artificial Intelligence (AI) encompass the development of algorithms and computer programs that can analyse data and make predictions in various fields.

▶ **Can Artificial Intelligence (AI) be used in the analysis of complex mathematical data to derive conclusions and facilitate decision-making accurately and swiftly?**

Issues and Life Skills

- Mathematical Communication   - Self-Confidence

- Language and Translation   - Creative Thinking

Values

- Responsibility   - Health Awareness

- Work   - Respecting Others

Get Ready!

Participation in school activities helps students in the development of their social skills, building self-confidence, and fostering creativity.

The opposite table illustrates two types of school activities: artistic activities and sports activities.

What are the all possible options for a participant to join in two activities, one is artistic and the other is sports?

School Activities	
Artistic Activities	Sports Activities
Music Art	Football Volleyball Basketball

In this lesson, you will learn the concept of a random experiment and the concept of an event, as well as how to find the sample space for a random experiment, which will enable you to solve such real-life problems.

Think & Discuss!

Sameh drew a ball from a basket containing 3 identical red balls without looking at it.	Magdy drew a ball from a basket containing 3 identical balls their colours are red, green, and yellow, without looking at it.	Samar drew a ball from a basket containing 3 identical balls of unknown colours without looking at it.

▶ Which of them, in your opinion, can determine the possible colour of the drawn ball before it is drawn?

▶ Which of them can definitively identify the colour of the drawn ball before it is drawn?

Learn!

Random Experiment – Sample Space

Random Experiment

A random experiment is any experiment for which all its possible outcomes can be determined before carrying it out, but you cannot predict which of these outcomes will actually occur when it is carried out.

Sample Space (Outcomes Space)

The sample space is the set of all possible outcomes of a random experiment, and it is usually denoted by the symbol (S), and the number of elements of the sample space is denoted by the symbol $n(S)$.

For example:
- The experiment of tossing a fair coin once and observing the face apparent is a random experiment because:
  - You cannot predict the actual outcome until the experiment is carried out.
  - You can determine all its possible outcomes before carrying it out, which are the appearance of a head (H) or a tail (T).
    The sample space is $\{H, T\}$, and it is written as $S = \{H, T\}$.
    The number of elements of the sample space is two, written as n $(S) = 2$.
- The experiment of selecting a card carrying the letter (B) from a set of identical cards, all carrying the letter (B), is not a random experiment, as its outcome can be definitely known before doing it, which is a card carrying the letter B.
- The experiment of drawing a coloured ball from a box containing a number of identical balls whose colours are unknown is not a random experiment, as it is not possible to predict the colour of the ball before carrying out the experiment.

■ Self-Evaluation ①

Indicate which of the following experiments is random and which is not, and then write the sample space for each of the random experiments, indicating the number of its elements:

❶ Drawing a ball from a bag containing a white ball, a yellow ball, a red ball, and a green ball, all identical, and observing its colour.

❷ Rolling a fair cube once, its faces carry the numbers from 30 up to 35 and observing the number appearing on the upper face.

❸ Drawing a card from 7 identical cards, all have the number 5, and observing the number written on the card.

Example 1

Indicate which of the following experiments is random and which is not, and then write the sample space for each of the random experiments, indicating the number of its elements:

❶ Rolling a fair die once and observing the number appearing on the upper face.

❷ Drawing a ball from a bag containing a red ball, a blue ball, and a white ball, all identical, and observing its colour.

❸ Drawing a ball from a set of identical green balls and observing the colour of the drawn ball.

❹ Drawing a card from 7 identical cards numbered from 12 to 18 and observing the number on the card.

❺ Drawing a numbered ball from a box containing a set of identical numbered balls (without knowing their numbers) and observing the number of the drawn ball.

✅

① Random experiment
  $S = \{1, 2, 3, 4, 5, 6\}$, $n(S) = 6$

② Random experiment, assuming: Red $= R$, Blue $= B$, White $= W$, then:
  $S = \{R, B, W\}$, $n(S) = 3$

③ Not a random experiment.

④ Random experiment.
  $S = \{12, 13, 14, 15, 16, 17, 18\}$, $n(S) = 7$

⑤ Not a random experiment.

Example 2

Write the sample space for each of the following random experiments, indicating the number of its elements:

① Experiment of tossing a fair coin twice and observing the sequence of heads and tails.

② Experiment of rolling a fair die twice and observing the number on the upper face in both rolls.

✅

① Each outcome of the experiment is an ordered pair, whose first component is the result of the first toss, and its second component is the result of the second toss. Since the possible outcomes for both tosses are: head (H) and tail (T), the opposite tree diagram can be used to find the elements of the sample space.

$S = \{(H, H), (H, T), (T, H), (T, T)\}$
$n(S) = 4$

▸ **Note that:** $(H, T) \neq (T, H)$

② Each outcome of the experiment is an ordered pair, whose first component is the result of the first roll, and its second component is the result of the second roll. The sample space (S) can be represented in tabular form or geometrically on a grid as follows:
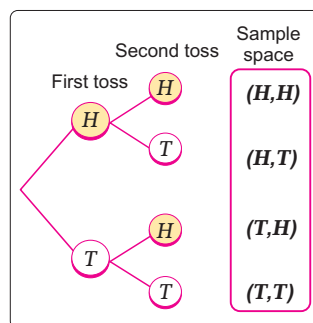
1 In tabular form

Second roll / First roll	1	2	3	4	5	6
1	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
2	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
3	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
4	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
5	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
6	(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

2 Geometrically on the grid
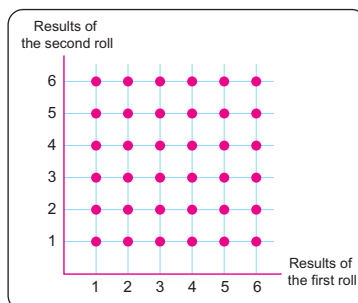
$n(S) = 6 \times 6 = 36$

Events

An event is a subset of the sample space.

An event is said to be occurred if the outcome of the random experiment after its carrying out is one of the elements of the set that form this event.

The certain event (S) is an event that must occur when carrying out the random experiment.

The impossible event (φ) is an event that cannot occur when carrying out the random experiment.

A simple event (or elementary event) is a subset of the sample space (S) that contains only one element.

A possible event is a proper subset of the sample space.

In the experiment of selecting a number from the numbers 2 to 11, write the sample space and then determine each of the following events, indicating which of them is simple, certain, or impossible:

1 Event of obtaining an odd number.

2 Event of obtaining a number less than 16.

3 Event of obtaining a number less than or equal to 4.

4 Event of obtaining the number 6.

5 Event of obtaining an even number divisible by 9.

6 Event of obtaining a perfect square number.

In the experiment of rolling a fair die twice and observing the number that appears on the upper face in both rolls, write each of the following events:

1 Event (A) is the event of obtaining two numbers their sum is 8.

2 Event (B) is the event of obtaining two numbers, the larger of them is the number 3.

3 Event (C) is the event of obtaining two equal numbers.

Example 3

If a fair die is rolled once and the number appearing on the upper face is observed, write the sample space and then determine each of the following events, indicating which of them is simple, certain, or impossible:

1 Event (A) is the event of obtaining an even number.

2 Event (B) is the event of obtaining a number greater than 1.

3 Event (C) is the event of obtaining an even prime number.

4 Event (D) is the event of obtaining a number less than 7.

5 Event (E) is the event of obtaining the number 8.

✅

The sample space is: $S = \{1, 2, 3, 4, 5, 6\}$

① $A = \{2, 4, 6\}$

② $B = \{2, 3, 4, 5, 6\}$

③ $C = \{2\}$ "Simple event"

④ $D = \{1, 2, 3, 4, 5, 6\} = S$ "Certain event"

⑤ $E = \phi$ "Impossible event"

Example 4

From the set of numbers $\{3, 4, 6, 7\}$, form a 2-digit number with different digits. Write the sample space for this experiment and then determine each of the following events:

1 Event (A) is the event of "the tens digit is odd."

2 Event (B) is the event of "the number is divisible by 4."

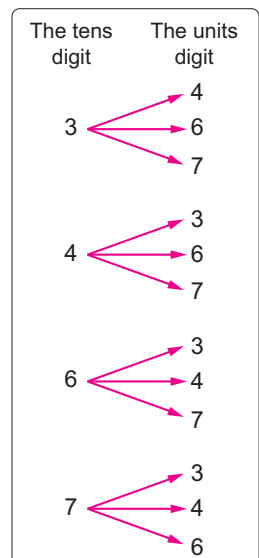3 Event (C) is the event of "the sum of the two digits is 10."

✅

The sample space is:

$S = \{34, 36, 37, 43, 46, 47, 63, 64, 67, 73, 74, 76\}$

① $A = \{34, 36, 37, 73, 74, 76\}$

② $B = \{36, 64, 76\}$

③ $C = \{37, 46, 64, 73\}$

The tens digit	The units digit
3	4
	6
	7
4	3
	6
	7
6	3
	4
	7
7	3
	4
	6

Lesson Assessment

▶ **Choose the correct answer from the given ones:**

1 Drawing a card from a set of identical numbered cards without knowing the numbers written on the cards:
(a) A random experiment.
(b) Not a random experiment.
(c) An impossible event.
(d) A certain event.

2 In the experiment of choosing a digit of the number 5742 randomly, what is the sample space?
(a) $\{2, 4, 5\}$ (b) $\{2, 4, 5, 7\}$
(c) $\{57, 74, 42\}$ (d) $\{5742\}$

3 In the experiment of forming a two-digit number of different digits from the set of digits $\{1, 3, 4\}$, how many elements are in the event that expresses that "the resulting number is odd"?
(a) 2   (b) 3   (c) 4   (d) 6

4 In the experiment of tossing a fair coin four consecutive times, how many elements are in the sample space?
(a) 2   (b) 4   (c) 8   (d) 16

5 In the experiment of rolling a fair die once, which of the following events is a simple event?
(a) Event of getting a number greater than 6.   (b) Event of getting an even prime number.
(c) Event of getting a number less than or equal to 2.   (d) Event of getting an odd prime number.

Second **Applying Scientific Concepts**

6 In the experiment of rolling a fair die once and observing the number that appears on the upper face, **write the sample space and then write each of the following events, indicating which of them is simple, certain, or impossible:**
① Event (A) is the event of getting a number greater than zero
② Event (B) is the event of getting a number divisible by 3
③ Event (C) is the event of getting a number less than or equal to 4
④ Event (D) is the event of getting a number that satisfies the inequality: $x > 5$
⑤ Event (E) is the event of getting an odd number that is not prime.
⑥ Event (F) is the event of getting a number greater than 4 and less than 5
⑦ Event (G) is the event of getting a number that is not a perfect square

7 A bag contains 25 identical cards numbered from 1 to 25, one card is drawn randomly, and the number on the drawn card is recorded. Write each of the following events:
① Event (A) is the event of drawing a number less than 4.
② Event (B) is the event of drawing a number that is a multiple of 6.
③ Event (C) is the event of drawing an odd number that is divisible by 5.
④ Event (D) is the event of drawing a perfect cube number.

⑧ In the experiment of tossing a fair coin twice in successively and observing the sequence of appearances of heads and tails, **write the sample space (S) and then express each of the following events:**

① Event (A) is the event of "appearance of a tail on the first toss."

② Event (B) is the event of "appearance of a tail on only one of the two tosses."

③ Event (C) is the event of "appearance of the same outcome on both tosses."

④ Event (D) is the event of "no heads appear."

⑤ Event (E) is the event of "appearance of different outcomes on both tosses."

⑨ A fair coin was tossed and then a fair die was rolled, and the upper face of the coin and the number appearing on the upper face of the die were observed.

Represent the sample space in a tree diagram, then find the following events:

① Event (A) is the event of "appearance of a tail and an even number."

② Event (B) is the event of "appearance of a head and an odd number."

⑩ In the experiment of rolling a fair die twice successively, write the following events:

① Event (A) is the event of "appearance of the number 3 on the second roll."

② Event (B) is the event of "appearance of two numbers whose sum is greater than or equal to 10."

③ Event (C) is the event of "appearance of two numbers whose sum is 15."

Third — **Analysis and Subjects Integration**

⑪ **Sports:** Upon joining one of the youth centers to participate in sports activities during the summer holiday, the sample space = {swimming, squash, volleyball, tennis, cycling, football}. If event A is participating in one of the ball games, and event B is participating in one of the racket games, write both events A and B.

Make sure to spend your holiday engaging in beneficial activities such as: sports and reading.

⑫ **Nutrition:** A restaurant offers lunch meals consisting of one main dish and one type of appetizer.
What are the different possibilities for choosing one meal?

Menu

Main Dish	Appetizer
Meat	Salad
Chicken	Soup
Fish	

Healthy food is essential for maintaining the body's health, as it strengthens the immune system and prevents diseases.

Evaluate your understanding!

How well do you understand random experiment - sample space - events? Tick the right box.

Theoretical & Experimental Probability

Get Ready!

The coach of the Egyptian national football team trained two players on taking penalty kicks. The first player took 12 penalty kicks, scoring 9 goals, while the second player took 15 penalty kicks, scoring 12 goals. If the team has the opportunity to take a penalty kick during one of the matches, which player will the coach choose to execute this penalty kick?

In this lesson, you will learn the concept of probability and how to find its value, which will enable you to solve such real-life problems.

Think & Discuss!

Each of Mohamed, Mariam, and Majid designed a spinning disc for a game, each disc is divided into circular sectors with equal areas as follows:
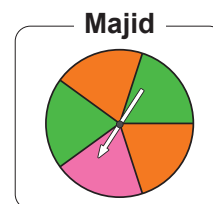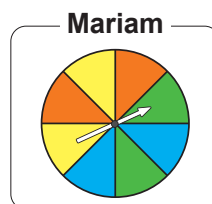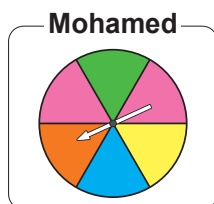
The pointer on each disc stops at one of the coloured sectors when spinning the disc.

▶ **Which of these discs do you think achieves the principle of equal opportunities?**

Learn!

Theoretical Probability

Theoretical probability is based on the principle of equal opportunities or equal possibilities, and it is equal to the ratio between the number of outcomes of the event to the total number of outcomes.

i.e. The probability of any event $(A) = \dfrac{\text{Number of outcomes of event } A}{\text{Total number of outcomes}}$

The probability of event A is denoted by the symbol $P(A)$.

Note that:

- The probability of an impossible event equals zero.
  This is written as: $P(\phi) = 0$

- The probability of a certain event equals one.
  This is written as: $P(S) = 1$

- The probability of any possible event lies between zero and one, as illustrated as follows:

The likelihood of occurrence decreases as probability approaches 0	Probability of occurrence as opposed to probability of non-occurrence	The likelihood of occurrence increases as probability approaches 1

Impossible   Certain

0 $\frac{1}{2}$ 1

0%   50%   100%

■ **Self-Evaluation** ①

A card was drawn randomly from a set of identical cards numbered from 5 to 14. Find the probability that the drawn card carries:
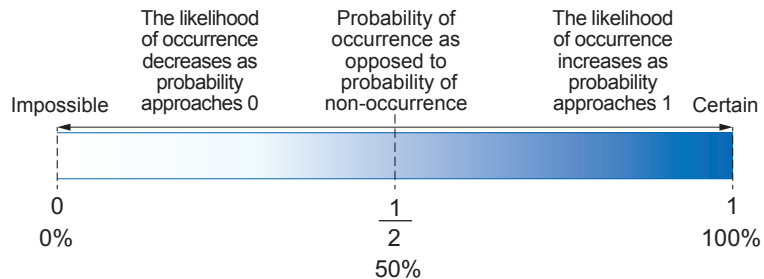
① An odd number.

② An even number greater than 9.

③ A prime number.

④ A number less than 5.

⑤ A perfect square number.

■ **Note that**

Probability can be written as a proper fraction, a decimal fraction, or a percentage.

Example 1

When a fair die is rolled once and the upper face is observed, find the probability of each of the following events:

① A the event of "getting an even number"

② B the event of "getting a number less than 8"

③ C the event of "getting a prime odd number"

④ D the event of "getting the number 4"

⑤ E the event of "getting a number greater than 6"

✅

All possible outcomes are: 1, 2, 3, 4, 5, 6, which are 6 outcomes.

① ∵ The even numbers are: 2, 4, 6, which are 3 outcomes.
$$\therefore P(A) = \frac{3}{6} = \frac{1}{2}$$
This can be written as: $P(A) = \frac{1}{2}$ **or** $P(A) = 0.5$ **or** $P(A) = 50\%$

---

② The numbers less than 8 are: 1, 2, 3, 4, 5, 6, which are 6 outcomes.
$$\therefore P(B) = \frac{6}{6} = 1$$
This can be written as: $P(B) = 1$ **or** $P(B) = 100\%$

---

③ The prime odd numbers are: 3, 5, which are 2 outcomes.
$$\therefore P(C) = \frac{2}{6} = \frac{1}{3}$$
This can be written as: $P(C) = \frac{1}{3}$ **or** $P(C) = 0.\overline{3}$ **or** $P(C) = 33\frac{1}{3}\%$

---

④ The number 4 is only one number.
$$\therefore P(D) = \frac{1}{6}$$

⑤ ∵ There is no number greater than 6, which means their total equals 0.
$$\therefore P(E) = \frac{0}{6} = 0$$

Example 2

A fair coin was tossed twice consecutively, and the sequence of heads and tails was observed. Find the probability of each of the following events:

❶ A the event of "getting two heads".

❷ B the event of "getting at least one head".

❸ C the event of "getting the same outcome in both tosses".

❹ D the event of "getting a head on the first toss".

✅

All possible outcomes are: (H, H), (H, T), (T, H), (T, T) which are 4 outcomes.

① The outcome with two heads is (H, H) and it is 1 outcome.

$\therefore P(A) = \dfrac{1}{4}$

② The outcomes with at least one head are (H, H), (H, T), (T, H) which are 3 outcomes.

$\therefore P(B) = \dfrac{3}{4}$

③ The outcomes with the same outcome in both tosses are (H, H), (T, T) which are 2 outcomes.

$\therefore P(C) = \dfrac{2}{4} = \dfrac{1}{2}$

④ The outcomes with a head on the first toss are (H, H), (H, T) which are 2 outcomes.

$\therefore P(D) = \dfrac{2}{4} = \dfrac{1}{2}$

Example 3

A bag contains one red ball, six blue balls, and three green balls, all of which are identical. If a ball is drawn randomly from the bag and its colour is observed, what is the probability that the drawn ball is:

❶ Blue? ❷ White?

❸ Red? ❹ Green?

❺ Blue or green? ❻ Not green?

✅
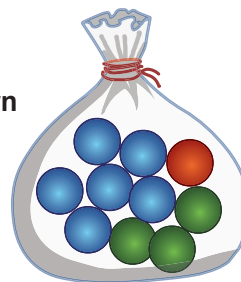
Assuming (Red $= R$), (Blue $= B$), (Green $= G$), (White $= W$)
The total number of balls $= 1 + 6 + 3 = 10$ balls.

① $P(B) = \dfrac{6}{10} = 0.6$ ② $P(W) = \dfrac{0}{10} = 0$

③ $P(R) = \dfrac{1}{10} = 0.1$ ④ $P(G) = \dfrac{3}{10} = 0.3$

⑤ $P(B \text{ or } G) = \dfrac{6+3}{10} = \dfrac{9}{10} = 0.9$

⑥ $P(\text{not } G) = \dfrac{1+6}{10} = \dfrac{7}{10} = 0.7$

▶ Another solution to calculate the probability that the drawn ball is not green:

$\because P(G) = 0.3$ $\therefore P(\text{not } G) = 1 - 0.3 = 0.7$

A fair die was rolled 150 times and the number appearing on the upper face was observed. The results of the appearances of the numbers are as follows:

Number	Number of times of appearance
1	28
2	19
3	23
4	28
5	25
6	27

▶ Find the experimental probability of:

❶ Appearing the number 2.

❷ Not appearing the number 5.

▶ Find the theoretical probability of appearing the number 2.

When a coloured ball is drawn from a bag containing four identical balls, red, green, blue, and white, the percentage of the number of times the ball was drawn is as shown in the following table:

Colour	Percentage of draws
Green	23%
Blue	27%
White	28%
Red	22%

❶ Find the experimental probability of not drawing the blue ball.

❷ If the number of times the experiment is carried out is 200, find the number of times a white ball is drawn.

❸ Find the theoretical probability of drawing the red ball.

Experimental Probability

Experimental probability is based on carrying out an experiment practically, recording its results, and then using these results to calculate the probability as follows:

Experimental probability of event $(A) = \dfrac{\text{Number of times event } (A) \text{ occurs}}{\text{Number of times the experiment is carried out}}$

Example 4

If a fair coin was tossed 100 times and head appeared 41 times, find the experimental probability of appearing:

❶ head (H) .　　　　❷ tail (T) .

✅

① The number of times the head (H) appeared is 41 times.
$\therefore P(H) = \dfrac{41}{100} = 0.41 = 41\%$

② The number of times the tail (T) appeared $= 100 - 41 = 59$
$\therefore P(T) = \dfrac{59}{100} = 0.59 = 59\%$

■ **Note that**

In the experiment of tossing a fair coin once, the theoretical probability of head appearing is $\dfrac{1}{2}$ ($= 50\%$), thus there is a difference between the experimental probability of head appearing in this example (41%) and the theoretical probability of head appearing (50%). It is observed that as the number of times of carrying out the experiment increases, the value of the experimental probability approaches the value of the theoretical probability.

Example 5

A spinning disc was divided into some coloured sectors equal in area. If the disc is spun 50 times, the opposite table shows the number of times the pointer stopped on each colour.
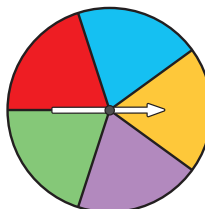
Colour	Number of times
Red	8
Blue	9
Yellow	13
Green	9
Purple	11

❶ Find the experimental probability of the pointer stopping on the yellow colour.
❷ Find the theoretical probability of the pointer stopping on the yellow colour.
❸ If the number of spins of the disc increases to 500 times, what do you expect regarding the chance of the pointer stopping on the yellow colour?

✅

① The experimental probability of the pointer stopping on the yellow colour is:
$\dfrac{13}{50} = 0.26 = 26\%$

② ∵ The five colours are equally distributed on the spinning disc.
∴ The theoretical probability of the pointer stopping on the yellow colour is:
$\dfrac{1}{5} = 0.2 = 20\%$

③ When the number of spins of the disc increases to 500 times, it is expected that the chance of the pointer stopping on the yellow colour decreases, till the value of the experimental probability approaches the value of the theoretical probability (20%).

Lesson Assessment

▶ **Choose the correct answer from the given ones:**

① If you are thinking of purchasing one pen from a set of identical pens containing 5 red pens, 2 blue pens, and 3 black pens, and you select this pen randomly what is the probability that the pen is blue?

(a) $\dfrac{1}{4}$ (b) $\dfrac{1}{5}$ (c) $\dfrac{2}{15}$ (d) $\dfrac{1}{15}$

② In an experiment of rolling a fair die once, what is the probability of getting a number divisible by 2?

(a) Zero   (b) $33\dfrac{1}{3}$%   (c) 50%   (d) 75%

③ When rolling a fair die 10 times consecutively, if the number 4 appears twice on the upper face of the die, what is the experimental probability of not appearing of the number 4?

(a) $\dfrac{1}{6}$ (b) $\dfrac{2}{10}$ (c) $\dfrac{5}{6}$ (d) $\dfrac{8}{10}$

④ If A is an event from a random experiment with equal chances of occurrence, and the probability of event A is 40%, the number of elements of the sample space is 15, what is the number of elements of event A?

(a) 2   (b) 4   (c) 6   (d) 10

⑤ Hamza has a spinning game divided into 9 equal sections, as illustrated in the opposite figure. When it spins, the pointer randomly stops on one of the sections. What is the probability that the pointer stops on blue or yellow?

(a) $\dfrac{2}{9}$ (b) $\dfrac{4}{9}$ (c) $\dfrac{7}{9}$ (d) $\dfrac{8}{9}$
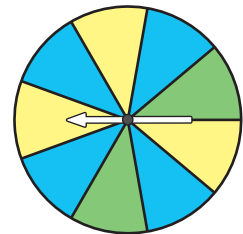
⑥ A classroom contains 15 students, 4 of them have black hair, 5 have brown hair, and 6 have blonde hair. If a student is chosen at random, find the probability that the student:

① has black hair.   ② does not have brown hair.   ③ has either blonde or brown hair.

⑦ In an experiment of rolling a fair die once, find the probability of each of the following events:

① A the event of getting an odd number.

② B the event of getting a number greater than 4.

③ C the event of getting the number 3.

④ D the event of getting a number less than 7.

⑤ E the event of getting a number that satisfies the inequality "$X \geq 2$".

⑥ F the event of getting a factor of the number 6.

⑧ If a card is drawn at random from identical cards numbered from 20 to 29, find the probability that the card carries a number:

① greater than 25.   ② less than 20.   ③ prime.   ④ even.

⑨ If you randomly select a number from the set of numbers {13, 17, 19, 23, 29, 31}, find the probability that the sum of the digits of the selected number is an even number.

⑩ In an experiment of forming a 2-digit number from the set of digits {4, 5, 7}, what is the probability of each of the following events:

① A the event that the sum of the two digits is 9? ② B the event that the tens digit is odd?

③ C the event that the product of the two digits is 35? ④ D the event that the tens digit equals the units digit?

⑪ **Culture:** In your personal library, you have 12 literary books, 8 historical books, and 10 scientific books. If you randomly choose a book, what is the probability that it is a literary book?


Reading is a way of acquiring knowledge, developing thought, and broadening horizons.

Fair competition is built upon respecting the rules of the game, adhering to sportsmanship, and respecting the opponent.

⑫ **Sports:** In an experiment of testing two players for joining in a basketball team at a club, the first player attempted 15 shots, scoring 6, while the second player attempted 20 shots, scoring 9. Determine which player the coach would select for the team, and why?

⑬ **Sports:** The opposite bar graph illustrates the preferred sports of 1000 students. If a student is chosen at random:

① What is the probability that this student prefers basketball?

② What is the probability that this student does not prefer swimming?

③ What is the probability that this student prefers football?
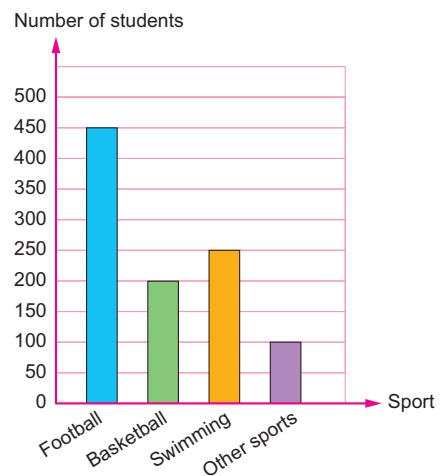
⑭ **Entertainment and Learning:**

The opposite stem-and-leaf plot illustrates the number of hours 30 students spend studying each week. If one student is chosen at random, what is the probability that the chosen student:

① Studies more than 32 hours?

② Studies less than 27 hours?

③ Studies more than 16 hours and less than 30 hours?

Stems	Leaves
0	1 4 5 6 8 8 9
1	0 0 1 1 1 6 7 8
2	0 1 2 2 3 4 5 5
3	0 1 3 4 5 6 6

Key | 1 | 6 means 16

Creative Thinking

⑮ A bag contains a number of identical balls, including 5 white balls and the remainder are red. If the probability of drawing a red ball equals $\frac{2}{3}$, find the total number of balls.

Evaluate *your understanding!*

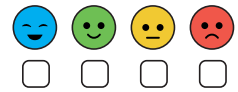How well do you understand theoretical & experimental probability?
Tick the right box

Unit Four Assessment

▶ **Choose the correct answer from the given ones:**

① In an experiment of rolling a fair die once, what is the probability of appearing a number less than 5?

(a) $\dfrac{1}{3}$ (b) $\dfrac{1}{2}$ (c) $\dfrac{2}{3}$ (d) $\dfrac{1}{6}$

② In an experiment of tossing a fair coin three consecutive times and observing the upper face, how many elements are there in the sample space?

(a) 2   (b) 4   (c) 8   (d) 16

③ A ball is drawn randomly from a box containing 35 identical balls, of which 7 are white and the remainder are red and black. What is the probability that the drawn ball is not white?

(a) $\dfrac{1}{35}$ (b) $\dfrac{1}{5}$ (c) $\dfrac{4}{5}$ (d) $\dfrac{34}{35}$

④ A card carring a letter from the word (Norhan) is drawn randomly. What is the probability that this letter is "N"?

(a) $\dfrac{1}{6}$ (b) $\dfrac{1}{3}$ (c) $\dfrac{2}{5}$ (d) $\dfrac{2}{3}$

▶ **Complete each of the following with the correct answer :**

⑤ When a card is drawn randomly from a set of identical cards numbered from 25 to 34, the event of appearing a number divisible by 3 on the drawn card is ············

⑥ A box contains 48 oranges, of which 6 are bad. If an orange is drawn randomly from the box, the probability that this orange is not bad = ············

⑦ A cube is designed such that it has two faces carring the number 2, two faces carring the number 4, and two faces carring the number 6. When the cube is rolled once and the number on the upper face is observed, the probability that the number on the upper face is prime is ············

⑧ When a fair die is rolled twice consecutively and the numbers on the upper faces are observed, the event that the sum of the two numbers equal 5 is ············

▶ **Answer the following questions :**

⑨ A bag contains 40 identical marbles. If Hani randomly draws a marble and he finds it red, and the probability of drawing a red marble equals $\dfrac{3}{5}$, find the number of red marbles in the bag.

⑩ Khaled has a spinning game divided into 8 equal sections, as illustrated in the opposite figure. When it spins, the pointer lands randomly on one section. Find each of the following:

  ① The probability that the pointer lands on a number greater than or equal to 4.

  ② The probability that the pointer lands on a number divisible by 6.

⑪ From the set of numbers {3, 4, 5}, form a 2-digit number and then find the probability of each of the following events:

  ① *A*: The event that the units digit is odd.

  ② *B*: The event that the sum of the two digits is 8.
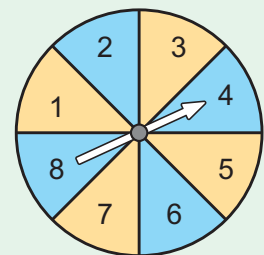
12 When drawing a card randomly from 6 identical cards carrying the following polygons:
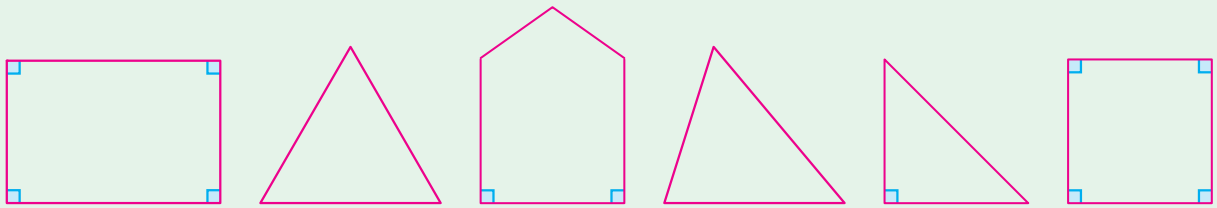
Find the probability that the card carries:

① A triangle.

② A polygon that is not a quadrilateral.

③ A polygon with more than three sides.

④ A polygon with more than one right angle.

⑤ A pentagon.

## Unit Four **Activity** **Who is the Potential Winner**

▶ **Activity Aim:**

To enhance students' understanding of the concepts of theoretical probability and experimental probability, in a practical manner through the spinning wheel game.

▶ **Implementation Steps:**

Collaborate with (3) of your classmates, carry out the steps of the following game and complete the following table to attempt to determine the winner of the game:

| Player	Result of the Spinning Wheel			
	Player (1)	Player (2)	Player (3)	Player (4)
Player (1)				
Player (2)				
Player (3)				
Player (4)				
Total				
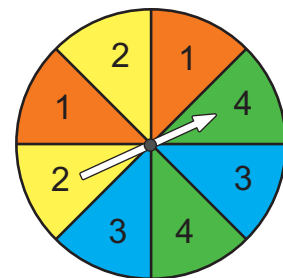
❶ Each individual spins the wheel (5 times) and identifies the player number at which the pointer stops.

❷ Use tally marks to record the result for the player number that appears in each attempt.

❸ Collaborate with your classmates, to answer the following questions:

- What is the total number of attempts that were made?

- What type of probability does this game represent?

☐ Theoretical probability   ☐ Experimental probability

- What is the probability that the pointer stops at the number of each player based on the results of the table?

❹ According to the shape of the spinning wheel and the results recorded in the table, did all four team members have equal chances of winning? Yes ☐   No ☐

Justification: ···

Evaluate your understanding!

How well do you understand the lessons of first preparatory? Tick the right box
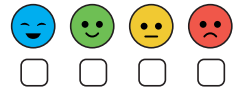
▶ **Choose the correct answer from the given ones:**

① In an experiment of tossing a fair coin two consecutive times, what is the number of times of appearance of one head at least?

(a) 1   (b) 2   (c) 3   (d) 4

② $\frac{1}{4}$ of the number 4^8 is ⋯⋯⋯⋯

(a) 4^2 (b) 4^4 (c) 4^6 (d) 4^7

③ A trapezium with a height of 5.4 cm and the lengths of its parallel bases are 8 cm and 10 cm, has an area of ⋯⋯⋯⋯ square centimeters.

(a) 48.6   (b) 54   (c) 97.2   (d) 432

④ If $7.5 \times 10^n = 0.000075$, what is the value of n?

(a) -5 (b) -4 (c) 4   (d) 5

⑤ $(X^3 + X^2 + X) \div X =$ ⋯⋯⋯⋯

(a) $X^3 + X^2$ (b) $X^2 + X$ (c) $X^2 + X + 1$ (d) 0

⑥ What is the image of the point (3 , 4) by translation $(X , Y) \longrightarrow (X - 4 , Y - 2)$?

(a) (2 , 1)   (b) (1 , -2)   (c) (-1 , 2)   (d) (-1 , -2)

⑦ If $X^3 + 124 = -1$, what is the value of X?

(a) -5 (b) -4 (c) 4   (d) 5

⑧ What is the image of the point (-2 , 4) by reflection in the X-axis?

(a) (-2 , -4)   (b) (2 , 4)   (c) (-4 , 2)   (d) (4 , 2)

⑨ A rhombus with diagonal lengths of 10 cm and 15 cm has an area of ⋯⋯⋯⋯ square centimeters.

(a) 37.5   (b) 75   (c) 150   (d) 300

▶ **Answer the following questions:**

① Draw a line segment of length 4.5 cm, then bisect it using a ruler and compass.

② Simplify to its simplest form: $(\frac{14}{15})^0 - \sqrt{\frac{9}{25}} + \sqrt[3]{\frac{64}{125}}$

③ If the quotient of the expression: $(X^3 - 25X)$ divided by $(X + 5)$ is $X^2 + aX$, what is the value of a?

④ Simplify the expression: $(4n - 3)^2 - (4n - 3)(4n + 3)$ to its simplest form, then find the numerical value of the expression when $n = -1$.

⑤ Draw on the grid the rectangle $ABCD$ where $A(1 , 1)$, $B(3 , 1)$, $C(3 , 6)$, $D(1 , 6)$, then find its image by rotation $R(O , 90°)$.

⑥ What is the solution set of the inequality $3X - 2 \leq 4$ in N?

⑦ In an experiment of rolling a fair die once, what is the probability of obtaining:

(A) a number greater than 2?   (B) a prime number less than 4?

MATHEMATICS

First Preparatory
Second Term

2024 - 2025

ألوان الغلاف	وزن الغلاف	ألوان المتن	وزن المتن	المقاس
4 لون	180 جم كوشيه	4 لون	70 جم ورق أبيض	19 × 27 سم